

**THE DYNKIN DIAGRAMS PACKAGE**  
**VERSION 3.14159265**

BEN MCKAY

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## 1. QUICK INTRODUCTION

Load the Dynkin diagram package (see options below)

```
\documentclass{amsart}
\usepackage{dynkin-diagrams}
\begin{document}
The Dynkin diagram of  $(B_3)$  is  $\text{dynkin}\{B\}\{3\}$ .
\end{document}
```

Invoke it

The Dynkin diagram of  $(B_3)$  is  $\text{dynkin}\{B\}\{3\}$ .

The Dynkin diagram of  $B_3$  is  $\bullet \rightarrow \bullet \rightarrow \bullet$ .

Inside a *TikZ* statement

The Dynkin diagram of  $(B_3)$  is  
 $\text{tikz } \text{dynkin}\{B\}\{3\};$

The Dynkin diagram of  $B_3$  is  $\bullet \rightarrow \bullet \rightarrow \bullet$

Inside a Dynkin diagram environment

The Dynkin diagram of  $(B_3)$  is  
 $\begin{array}{l} \text{begin}\{\text{dynkinDiagram}\}\{B\}\{3\} \\ \text{draw[very thick,red]} (\text{root } 1) \text{ to } [\text{out}=-45, \text{in}=-135] (\text{root } 3); \\ \text{end}\{\text{dynkinDiagram}\} \end{array}$

The Dynkin diagram of  $B_3$  is  $\bullet \rightarrow \bullet \rightarrow \bullet$

Inside a *TikZ* environment

Baseline controls vertical alignment:  
the Dynkin diagram of  $(B_3)$  is  
 $\begin{array}{l} \text{begin}\{\text{tikzpicture}\}[\text{baseline}=(\text{origin.base})] \\ \text{dynkin}\{B\}\{3\} \\ \text{draw[very thick,red]} (\text{root } 1) \text{ to } [\text{out}=-45, \text{in}=-135] (\text{root } 3); \\ \text{end}\{\text{tikzpicture}\} \end{array}$

Baseline controls vertical alignment: the Dynkin diagram of  $B_3$  is  $\bullet \rightarrow \bullet \rightarrow \bullet$

## Indefinite rank Dynkin diagrams

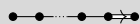
`\dynkin{B}{}`

Table 1: The Dynkin diagrams of the reduced simple root systems  
[3] pp. 265–290, plates I–IX

$A_n$		<code>\dynkin{A}{}</code>
$C_n$		<code>\dynkin{C}{}</code>
$D_n$		<code>\dynkin{D}{}</code>
$E_6$		<code>\dynkin{E}{6}</code>
$E_7$		<code>\dynkin{E}{7}</code>
$E_8$		<code>\dynkin{E}{8}</code>
$F_4$		<code>\dynkin{F}{4}</code>
$G_2$		<code>\dynkin{G}{2}</code>

## 2. SET OPTIONS GLOBALLY

Most options set globally ...

```
\pgfkeys{/Dynkin diagram,edge length=.5cm,fold radius=.5cm,
indefinite edge/.style={
draw=black,fill=white,thin,densely dashed}}
```

You can also pass options to the package in `\usepackage`. *Danger*: spaces in option names are replaced with hyphens: `edge length=1cm` is `edge-length=1cm` as a global option; moreover you should drop the extension `/.style` on any option with spaces in its name (but not otherwise). For example,

...or pass global options to the package

```
\usepackage[
ordering=Kac,
edge/.style=blue,
indefinite-edge={draw=green,fill=white,densely dashed},
indefinite-edge-ratio=5,
mark=o,
root-radius=.06cm]
{dynkin-diagrams}
```

## 3. COXETER DIAGRAMS

Coxeter diagram option

`\dynkin[Coxeter]{F}{4}`gonality option for  $G_2$  and  $I_n$  Coxeter diagrams
$$\backslash(G_2=\backslash\text{dynkin}[Coxeter,gonality=n]\{G\}\{2\}\backslash), \backslash$$

$$\backslash(I_n=\backslash\text{dynkin}[Coxeter,gonality=n]\{I\}\{\}\backslash)$$

$$G_2 = \overset{n}{\bullet} - \bullet, \quad I_n = \bullet - \overset{n}{\bullet}$$

Table 2: The Coxeter diagrams of the simple reflection groups

$A_n$		<code>\dynkin[Coxeter]{A}{}</code>
$B_n$		<code>\dynkin[Coxeter]{B}{}</code>
$C_n$		<code>\dynkin[Coxeter]{C}{}</code>
$E_6$		<code>\dynkin[Coxeter]{E}{6}</code>
$E_7$		<code>\dynkin[Coxeter]{E}{7}</code>
$E_8$		<code>\dynkin[Coxeter]{E}{8}</code>
$F_4$		<code>\dynkin[Coxeter]{F}{4}</code>
$G_2$		<code>\dynkin[Coxeter,gonality=n]{G}{2}</code>
$H_3$		<code>\dynkin[Coxeter]{H}{3}</code>
$H_4$		<code>\dynkin[Coxeter]{H}{4}</code>
$I_n$		<code>\dynkin[Coxeter,gonality=n]{I}{}</code>

## 4. SATAKE DIAGRAMS

Satake diagrams use the standard name instead of a rank

$\backslash(A_{IIIb}=\backslash\text{dynkin}\{A\}\{IIIb\}\backslash)$

$$A_{IIIb} = \begin{array}{c} \circ - \circ - \circ - \circ \\ | \quad | \quad | \quad | \\ \circ - \circ - \circ - \circ \end{array}$$

We use a solid gray bar to denote the folding of a Dynkin diagram, rather than the usual double arrow, since the diagrams turn out simpler and easier to read.

Table 3: The Satake diagrams of the real simple Lie algebras [13] p. 532–534

$A_I$		$\backslash\text{dynkin}\{A\}\{I\}$
$A_{II}$		$\backslash\text{dynkin}\{A\}\{II\}$
$A_{IIIa}$		$\backslash\text{dynkin}\{A\}\{IIIa\}$
$A_{IIIb}$		$\backslash\text{dynkin}\{A\}\{IIIb\}$
$A_{IV}$		$\backslash\text{dynkin}\{A\}\{IV\}$
$B_I$		$\backslash\text{dynkin}\{B\}\{I\}$
$B_{II}$		$\backslash\text{dynkin}\{B\}\{II\}$
$C_I$		$\backslash\text{dynkin}\{C\}\{I\}$
$C_{IIa}$		$\backslash\text{dynkin}\{C\}\{IIa\}$
$C_{IIb}$		$\backslash\text{dynkin}\{C\}\{IIb\}$
$D_{Ia}$		$\backslash\text{dynkin}\{D\}\{Ia\}$
$D_{Ib}$		$\backslash\text{dynkin}\{D\}\{Ib\}$
$D_{Ic}$		$\backslash\text{dynkin}\{D\}\{Ic\}$
$D_{II}$		$\backslash\text{dynkin}\{D\}\{II\}$
$D_{IIIa}$		$\backslash\text{dynkin}\{D\}\{IIIa\}$
$D_{IIIb}$		$\backslash\text{dynkin}\{D\}\{IIIb\}$
$E_I$		$\backslash\text{dynkin}\{E\}\{I\}$
$E_{II}$		$\backslash\text{dynkin}\{E\}\{II\}$

continued ...

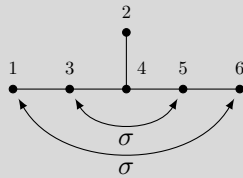
Table 3: ...continued

$E_{III}$		<code>\dynkin{E}{III}</code>
$E_{IV}$		<code>\dynkin{E}{IV}</code>
$E_V$		<code>\dynkin{E}{V}</code>
$E_{VI}$		<code>\dynkin{E}{VI}</code>
$E_{VII}$		<code>\dynkin{E}{VII}</code>
$E_{VIII}$		<code>\dynkin{E}{VIII}</code>
$E_{IX}$		<code>\dynkin{E}{IX}</code>
$F_I$		<code>\dynkin{F}{I}</code>
$F_{II}$		<code>\dynkin{F}{II}</code>
$G_I$		<code>\dynkin{G}{I}</code>

## 5. HOW TO FOLD

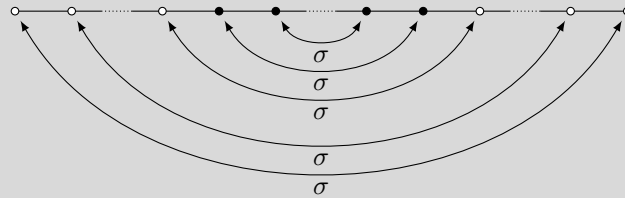
If you don't like the solid gray "folding bar", most people use arrows. Here is  $E_{II}$

```
\newcommand{\invol}[2]{\draw[latex-latex] (root #1) to
[out=-60,in=-120] node[midway,below]{ $\sigma$ } (root #2);}
\begin{dynkinDiagram}[edge length=.75cm,labels*={1,...,6}]{E}{6}
\invol{1}{6}\invol{3}{5}
\end{dynkinDiagram}
```



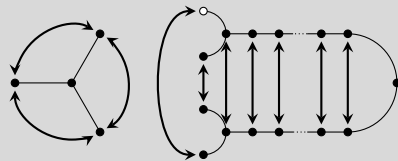
The double arrows for  $A_{IIIa}$  are big

```
\newcommand{\invol}[2]{\draw[latex-latex] (root #1) to
[out=-60,in=-120] node[midway,below]{\sigma} (root #2);}
\begin{dynkinDiagram}[edge length=.75cm]{A}{oo.o**.**.oo}
\invol{1}{10}\invol{2}{9}\invol{3}{8}\invol{4}{7}\invol{5}{6}
\end{dynkinDiagram}
```



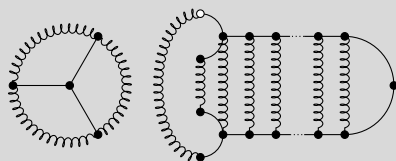
If you don't like the solid gray "folding bar", most people use arrows ...

```
\tikzset{/Dynkin diagram/fold style/.style={stealth-stealth,thick,
shorten <=1mm,shorten >=1mm,}}
\dynkin[ply=3,edge length=.75cm]{D}{4}
\begin{dynkinDiagram}[ply=4]{D}[1]%
{****.*****.*****}
\dynkinFold{1}{13}
\dynkinFold[bend right=90]{0}{14}
\end{dynkinDiagram}
```



...but you could try springs pulling roots together

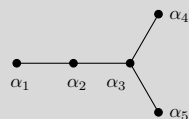
```
\tikzset{/Dynkin diagram/fold style/.style=
{decorate,decoration={name=coil,aspect=0.5,
segment length=1mm,amplitude=.6mm}}}
\dynkin[ply=3,edge length=.75cm]{D}{4}
\begin{dynkinDiagram}[ply=4]{D}[1]%
{****.*****.*****}
\dynkinFold{1}{13}
\dynkinFold[bend right=90]{0}{14}
\end{dynkinDiagram}
```



## 6. LABELS FOR THE ROOTS

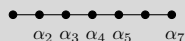
Make a macro to assign labels to roots

```
\dynkin[label,label macro/.code={\alpha_{\mathrlap{\#1}}}},edge
length=.75cm]{D}{5}
```



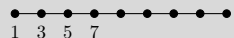
Labelling several roots

```
\dynkin[labels={2,...,5,,7},label
macro/.code={\alpha_{\mathrlap{\#1}}}}{A}{7}
```



The foreach notation I

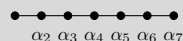
```
\dynkin[labels={1,3,...,7},]{A}{9}
```





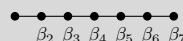
## The foreach notation II

```
\dynkin[labels={,\alpha_2,\alpha_....,\alpha_7},]{A}{7}
```



## The foreach notation III

```
\dynkin[label  
macro/.code={\beta_{\mathrlap{#1}}},labels={,2,...,7},]{A}{7}
```



## Label the roots individually by root number

```
\dynkin[label]{B}{3}
```



## Label a single root

```
\begin{dynkinDiagram}{B}{3}  
\dynkinLabelRoot{2}{\alpha_2}  
\end{dynkinDiagram}
```



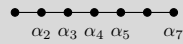
## Access root labels via TikZ

```
\begin{dynkinDiagram}{B}{3}  
\node[below] at (root 2) {\(\alpha_{\mathrlap{2}}\)};  
\end{dynkinDiagram}
```



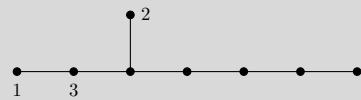
Commands to label several roots

```
\begin{dynkinDiagram}{A}{7}
\dynkinLabelRoots{\alpha_2,\alpha_3,\alpha_4,\alpha_5,,\alpha_7}
\end{dynkinDiagram}
```



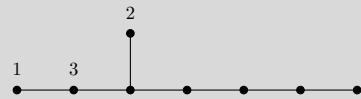
The labels have default locations, mostly below roots

```
\dynkin[edge length=.75cm,labels={1,2,3}]{E}{8}
```



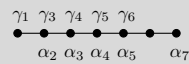
The starred form flips labels to alternate locations, mostly above roots

```
\dynkin[edge length=.75cm,labels*={1,2,3}]{E}{8}
```



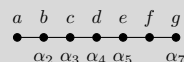
Labelling several roots and alternates

```
\dynkin[%
label macro/.code={\alpha_{\mathrlap{\#1}}},
label macro*/.code={\gamma_{\mathrlap{\#1}}},
labels={2,...,5,,7},
labels*={1,3,4,5,6}]{A}{7}
```



## Commands to label several roots

```
\begin{dynkinDiagram}{A}{7}
\dynkinLabelRoots{,\alpha_2,\alpha_3,\alpha_4,\alpha_5,,\alpha_7}
\dynkinLabelRoots*{a,b,c,d,e,f,g}
\end{dynkinDiagram}
```

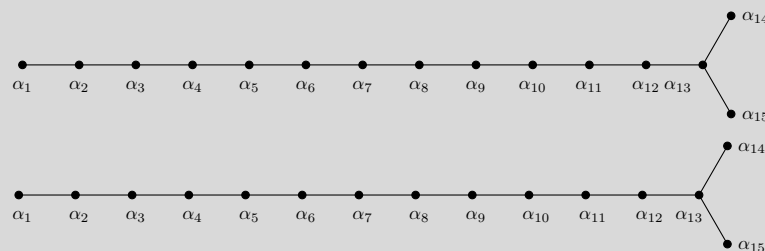


## 7. LABEL SUBSCRIPTS

Note the slight improvement that `\mathrlap` makes: the labels are centered on the middle of the letter  $\alpha$ , ignoring the space taken up by the subscripts.

## Label spacing improvement

```
\dynkin[label,label macro/.code={\alpha_{#1}},edge length=.75cm]{D}{15}
\par\noindent{}%
\dynkin[label,label macro/.code={\alpha_{\mathrlap{#1}}},edge
length=.75cm]{D}{15}
```

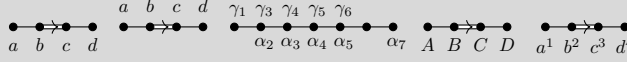


## 8. HEIGHT AND DEPTH OF LABELS

Labels are set with default maximum height the height of the character  $b$ , and default maximum depth the depth of the character  $g$ . To change these, set label height and label depth:

Change height and dept of characters

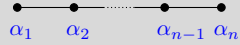
```
\dynkin[labels={a,b,c,d}]{F}{4}
\dynkin[labels*={a,b,c,d}]{F}{4}
\dynkin[%
label macro/.code={\alpha_{\mathrlap{#1}}},
label macro*/.code={\gamma_{\mathrlap{#1}}},
label height=$\alpha_1$,
label depth=$\alpha_1$,
labels={2,...,5,,7},
labels*={1,3,4,5,6}{A}{7}
\dynkin[labels={A,B,C,D},label height=$A$,label depth=$A$]{F}{4}
\dynkin[labels={a^1,b^2,c^3,d^4},label height=$X^X$]{F}{4}
```



## 9. TEXT STYLE FOR THE LABELS

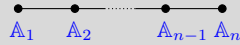
Use a text style: big and blue

```
\begin{dynkinDiagram}[text style={scale=1.2,blue},
edge length=.75cm,
labels={1,2,n-1,n},
label macro/.code={\alpha_{\mathrlap{#1}}}
]{A}{}
\end{dynkinDiagram}
```



Use a text style; font selection is in the label macro

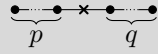
```
\begin{dynkinDiagram}[text style={scale=1.2,blue},
edge length=.75cm,
labels={1,2,n-1,n},
label macro/.code={\mathbb{A}_{\mathrlap{#1}}}]{A}{}
\end{dynkinDiagram}
```



## 10. BRACING ROOTS

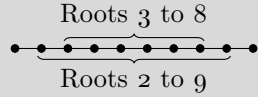
Bracing roots

```
\begin{dynkinDiagram}{A}{*.x*.*}
\dynkinBrace[p]{1}{2}
\dynkinBrace[q]{4}{5}
\end{dynkinDiagram}
```



Bracing roots, and a starred form

```
\begin{dynkinDiagram}{A}{10}
\dynkinBrace[\text{Roots 2 to 9}]{2}{9}
\dynkinBrace*[\text{Roots 3 to 8}]{3}{8}
\end{dynkinDiagram}
```



Bracing roots

```
\newcommand\circleRoot[1]{\draw (root #1) circle (3pt);}
\begin{dynkinDiagram}{A}{**.*.*.*.*.*.*.*}
\circleRoot{4}\circleRoot{7}\circleRoot{10}\circleRoot{13}
\dynkinBrace[y-1]{1}{3}
\dynkinBrace[z-1]{5}{6}
\dynkinBrace[t-1]{11}{12}
\dynkinBrace[x-1]{14}{16}
\end{dynkinDiagram}
```

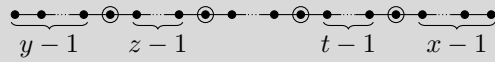
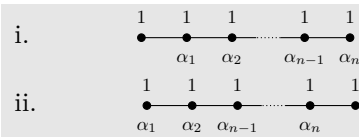


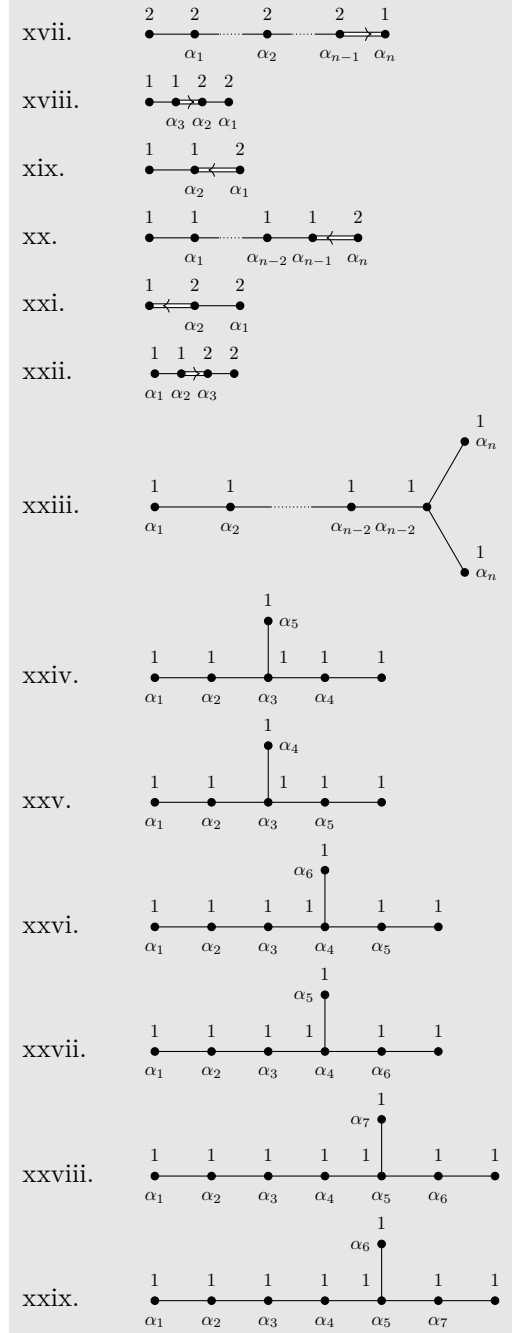
Table 4: Dynkin diagrams from Euler products [17]



continued ...

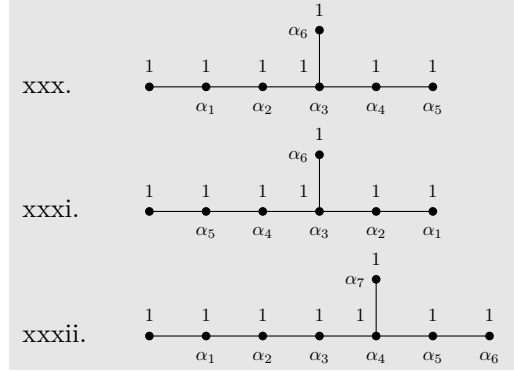


Table 4: ...continued



continued ...

Table 4: ...continued



```

\tikzset{/Dynkin diagram,ordering=Dynkin,label macro/.code={\alpha_{#1}}
\newcounter{EPNo}
\setcounter{EPNo}{0}
\NewDocumentCommand\EP{smmmm}%
{%
\stepcounter{EPNo}\roman{EPNo}. &
\def\eL{.6cm}
\IfStrEqCase{#2}%
{%
{D}{\gdef\eL{1cm}}%
{E}{\gdef\eL{.75cm}}%
{F}{\gdef\eL{.35cm}}%
{G}{\gdef\eL{.35cm}}%
}%
\tikzset{/Dynkin diagram,edge length=\eL}
\IfBooleanTF{#1}%
{\dynkin[backwards,labels*={#4},labels={#5}]{#2}{#3}}
{\dynkin[labels*={#4},labels={#5}]{#2}{#3}}
\\
}%
\begin{longtable}{MM}
\caption{Dynkin diagrams from Euler products \cite{Langlands:1967}}\\
\endfirsthead
\caption{\dots continued}\\
\endhead
\endhead
\multicolumn{2}{c}{continued \dots}\\
\endfoot
\endlastfoot
\EP{A}{***.***}{1,1,1,1,1}{1,2,n-1,n}
\EP{A}{***.***}{1,1,1,1,1}{1,2,n-1,n}
\EP{A}{**.*.*.}{1,1,1,1,1,1}{1,2,m-1,,m,n}
\EP{B}{**.*.*.}{2,2,2,2,1}{1,2,n-1,n}
\EP*{B}{***.***}{2,2,2,2,1}{n,n-1,2,1,}
\EP{C}{**.*.*.}{1,1,1,1,2}{1,2,n-1,}
\EP*{C}{***.***}{1,1,1,1,2}{n,n-1,2,1,}
\EP{D}{**.*.*.}{1,1,1,1,1,1}{1,2,n-2,n-1,n}
\EP{D}{**.*.*.}{1,1,1,1,1,1}{1,2,n-2,n-1,n}

```



```

\EP{E}{6}{1,1,1,1,1}{1,...,5}
\EP*{E}{7}{1,1,1,1,1,1}{6,...,1}
\EP{E}{7}{1,1,1,1,1,1}{1,...,6}
\EP*{E}{8}{1,1,1,1,1,1,1}{7,...,1}
\EP{E}{8}{1,1,1,1,1,1,1}{1,...,7}
\EP{G}{2}{1,3}{1}
\EP{G}{2}{1,3}{1}
\EP{B}{***.***}{2,2,2,2,1}{1,2,n-1,n}
\EP{F}{4}{1,1,2,2}{3,2,1}
\EP{C}{3}{1,1,2}{2,1}
\EP{C}{***.***}{1,1,1,2}{1,n-2,n-1,n}
\EP*{B}{3}{2,2,1}{1,2}
\EP{F}{4}{1,1,2,2}{1,2,3}
\EP{D}{***.***}{1,1,1,1,1}{1,2,n-2,n-2,n,n}
\EP{E}{6}{1,1,1,1,1,1}{1,2,3,4,5}
\EP{E}{6}{1,1,1,1,1,1}{1,2,3,5,4}
\EP*{E}{7}{1,1,1,1,1,1,1}{5,...,1,6}
\EP*{E}{7}{1,1,1,1,1,1,1}{6,4,3,2,1,5}
\EP*{E}{8}{1,1,1,1,1,1,1,1}{6,...,1,7}
\EP*{E}{8}{1,1,1,1,1,1,1,1}{7,5,4,3,2,1,6}
\EP*{E}{7}{1,1,1,1,1,1,1,1}{5,...,1,6}
\EP*{E}{7}{1,1,1,1,1,1,1,1}{1,...,5,6}
\EP*{E}{8}{1,1,1,1,1,1,1,1}{6,...,1,7}
\end{longtable}

```

## 11. STYLE

### Colours

```

\dynkin[
  edge/.style={blue!50,thick},
  */.style=blue!50!red,
  arrow color=red]{F}{4}

```



### Edge lengths

```

The Dynkin diagram of  $(A_3)$  is \dynkin[edge
length=1.2,parabolic=3]{A}{3}

```

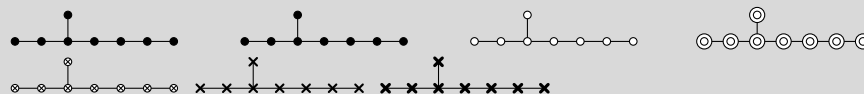
The Dynkin diagram of  $A_3$  is  $\times \text{---} \times \text{---} \bullet$

## Root marks

```

\dynkin{E}{8}
\dynkin[mark=*]{E}{8}
\dynkin[mark=o]{E}{8}
\dynkin[mark=O]{E}{8}
\dynkin[mark=t]{E}{8}
\dynkin[mark=x]{E}{8}
\dynkin[mark=X]{E}{8}

```



At the moment, you can only use:

- \* solid dot
- o hollow circle
- O double hollow circle
- t tensor root
- x crossed root
- X thickly crossed root

## Mark styles

The parabolic subgroup  $\backslash(E_{8,124})\backslash$  is  
`\dynkin[parabolic=124,x/.style={brown,very thick}]{E}{8}`

The parabolic subgroup  $E_{8,124}$  is

## Sizes of root marks

$\backslash(A_{3,3})\backslash$  with big root marks is `\dynkin[root radius=.08cm,parabolic=3]{A}{3}`

$A_{3,3}$  with big root marks is

## 12. SUPPRESS OR REVERSE ARROWS

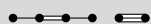
Some diagrams have double or triple edges

```
\dynkin{F}{4}
\dynkin{G}{2}
```



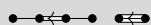
Suppress arrows

```
\dynkin[arrows=false]{F}{4}
\dynkin[arrows=false]{G}{2}
```



Reverse arrows

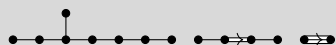
```
\dynkin[reverse arrows]{F}{4}
\dynkin[reverse arrows]{G}{2}
```



## 13. BACKWARDS AND UPSIDE DOWN

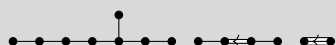
Default

```
\dynkin{E}{8}
\dynkin{F}{4}
\dynkin{G}{2}
```



Backwards

```
\dynkin[backwards]{E}{8}
\dynkin[backwards]{F}{4}
\dynkin[backwards]{G}{2}
```



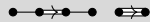
## Reverse arrows

```
\dynkin[reverse arrows]{F}{4}
\dynkin[reverse arrows]{G}{2}
```



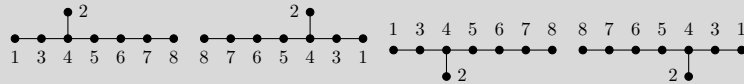
## Backwards, reverse arrows

```
\dynkin[backwards,reverse arrows]{F}{4}
\dynkin[backwards,reverse arrows]{G}{2}
```



## Backwards versus upside down

```
\dynkin[label]{E}{8}
\dynkin[label,backwards]{E}{8}
\dynkin[label,upside down]{E}{8}
\dynkin[label,backwards,upside down]{E}{8}
```



## 14. DRAWING ON TOP OF A DYNKIN DIAGRAM

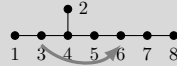
## TikZ can access the roots themselves

```
\begin{dynkinDiagram}{A}{4}
  \fill[white,draw=black] (root 2) circle (.15cm);
  \fill[white,draw=black] (root 2) circle (.1cm);
  \draw[black] (root 2) circle (.05cm);
\end{dynkinDiagram}
```



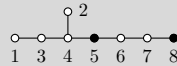
Draw curves between the roots

```
\begin{dynkinDiagram}[label]{E}{8}
  \draw[very thick, black!50,-latex]
    (root 3.south) to [out=-45, in=-135] (root 6.south);
\end{dynkinDiagram}
```



Change marks

```
\begin{dynkinDiagram}[mark=o,label]{E}{8}
  \dynkinRootMark{*}{5}
  \dynkinRootMark{*}{8}
\end{dynkinDiagram}
```



## 15. MARK LISTS

The package allows a list of root marks instead of a rank:

A mark list

```
\dynkin{E}{oo**ttxx}
```



The mark list `oo**ttxx` has one mark for each root: `o`, `o`, `.`, `.`, `x`, `x`, `x`, `x`. Roots are listed in the current default ordering. (Careful: in an affine root system, a mark list will *not* contain a mark for root zero.)

If you need to repeat a mark, you can give a *single digit* positive integer to indicate how many times to repeat it.

A mark list with repetitions

```
\dynkin{A}{x4o3t4}
```

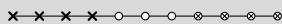


Table 5: Classical Lie superalgebras [10]. We need a slightly larger root radius parameter to distinguish the tensor product symbols from the solid dots.

		<code>\tikzset{/Dynkin diagram,root radius=.07cm}</code>
$A_{mn}$		<code>\dynkin{A}{o3.oto.oo}</code>
$B_{mn}$		<code>\dynkin{B}{o3.oto.oo}</code>
$B_{0n}$		<code>\dynkin{B}{o3.o3.o*}</code>
$C_n$		<code>\dynkin{C}{too.oto.oo}</code>
$D_{mn}$		<code>\dynkin{D}{o3.oto.o4}</code>
$D_{21\alpha}$		<code>\dynkin{A}{oto}</code>
$F_4$		<code>\dynkin{F}{ooot}</code>
$G_3$		<code>\dynkin[extended,affine mark=t,reverse arrows]{G}{2}</code>

Table 6: Classical Lie superalgebras [10]. Here we see the problem with using the default root radius parameter, which is too small for tensor product symbols.

$A_{mn}$		<code>\dynkin{A}{o3.oto.oo}</code>
$B_{mn}$		<code>\dynkin{B}{o3.oto.oo}</code>
$B_{0n}$		<code>\dynkin{B}{o3.o3.o*}</code>
$C_n$		<code>\dynkin{C}{too.oto.oo}</code>
$D_{mn}$		<code>\dynkin{D}{o3.oto.o4}</code>
$D_{21\alpha}$		<code>\dynkin{A}{oto}</code>
$F_4$		<code>\dynkin{F}{ooot}</code>
$G_3$		<code>\dynkin[extended,affine mark=t,reverse arrows]{G}{2}</code>

## 16. INDEFINITE EDGES

An *indefinite edge* is a dashed edge between two roots,  $\bullet \cdots \bullet$  indicating that an indefinite number of roots have been omitted from the Dynkin diagram. In between any two entries in a mark list, place a period to indicate an indefinite edge:

Indefinite edges

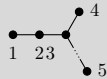
`\dynkin{D}{o.o*.*.t.to.t}`



In certain diagrams, roots may have an edge between them even though they are not subsequent in the ordering. For such rare situations, there is an option:

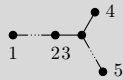
#### Indefinite edge option

```
\dynkin[make indefinite edge={3-5},label]{D}{5}
```



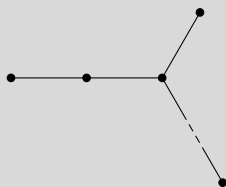
#### Give a list of edges to become indefinite

```
\dynkin[make indefinite edge/.list={1-2,3-5},label]{D}{5}
```



#### Indefinite edge style

```
\dynkin[indefinite edge/.style={draw=black,fill=white,thin,densely dashed},%
edge length=1cm,%
make indefinite edge={3-5}]
{D}{5}
```



#### The ratio of the lengths of indefinite edges to those of other edges

```
\dynkin[edge length = .5cm,%
indefinite edge ratio=3,%
make indefinite edge={3-5}]
{D}{5}
```

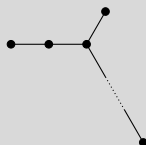


Table 7: Springer’s table of indices [24], pp. 320-321, with one form of  $E_7$  corrected

$A_n$		
$A_n$		
$B_n$		
$C_n$		
$D_n$		
$E_6$		$\backslash\mathrm{dynkin}\{\mathrm{E}\}\{*\mathrm{o}\mathrm{o}\mathrm{o}\mathrm{o}*\}$
$E_6$		$\backslash\mathrm{dynkin}\{\mathrm{E}\}\{\mathrm{o}*\mathrm{o}*\mathrm{o}\mathrm{o}\}$
$E_6$		$\backslash\mathrm{dynkin}\{\mathrm{E}\}\{\mathrm{o}*\mathrm{o}\mathrm{o}\mathrm{o}\mathrm{o}\}$
$E_6$		$\backslash\mathrm{dynkin}\{\mathrm{E}\}\{**\mathrm{o}\mathrm{o}\mathrm{o}*\}$
$E_7$		$\backslash\mathrm{dynkin}\{\mathrm{E}\}\{*\mathrm{o}\mathrm{o}\mathrm{o}\mathrm{o}\mathrm{o}\mathrm{o}\}$
$E_7$		$\backslash\mathrm{dynkin}\{\mathrm{E}\}\{\mathrm{o}\mathrm{o}\mathrm{o}\mathrm{o}\mathrm{o}*\mathrm{o}\}$
$E_7$		$\backslash\mathrm{dynkin}\{\mathrm{E}\}\{\mathrm{o}\mathrm{o}\mathrm{o}\mathrm{o}\mathrm{o}*\mathrm{o}\}$
$E_7$		$\backslash\mathrm{dynkin}\{\mathrm{E}\}\{*\mathrm{o}\mathrm{o}\mathrm{o}\mathrm{o}*\mathrm{o}\}$
$E_7$		$\backslash\mathrm{dynkin}\{\mathrm{E}\}\{*\mathrm{o}\mathrm{o}\mathrm{o}\mathrm{o}**\}$
$E_7$		$\backslash\mathrm{dynkin}\{\mathrm{E}\}\{*\mathrm{o}**\mathrm{o}*\mathrm{o}\}$
$E_8$		$\backslash\mathrm{dynkin}\{\mathrm{E}\}\{*\mathrm{o}\mathrm{o}\mathrm{o}\mathrm{o}\mathrm{o}\mathrm{o}\mathrm{o}\}$
$E_8$		$\backslash\mathrm{dynkin}\{\mathrm{E}\}\{\mathrm{o}\mathrm{o}\mathrm{o}\mathrm{o}\mathrm{o}\mathrm{o}*\}$
$E_8$		$\backslash\mathrm{dynkin}\{\mathrm{E}\}\{*\mathrm{o}\mathrm{o}\mathrm{o}\mathrm{o}\mathrm{o}*\}$
$E_8$		$\backslash\mathrm{dynkin}\{\mathrm{E}\}\{\mathrm{o}\mathrm{o}\mathrm{o}\mathrm{o}\mathrm{o}**\}$
$E_8$		$\backslash\mathrm{dynkin}\{\mathrm{E}\}\{*\mathrm{o}\mathrm{o}\mathrm{o}\mathrm{o}***\}$
$F_4$		$\backslash\mathrm{dynkin}\{\mathrm{F}\}\{\mathrm{o}\mathrm{o}\mathrm{o}*\}$
$D_4$		$\backslash\mathrm{dynkin}\{\mathrm{D}\}\{\mathrm{o}*\mathrm{o}\mathrm{o}\}$



## 17. PARABOLIC SUBGROUPS

Each set of roots is assigned a number, with each binary digit zero or one to say whether the corresponding root is crossed or not:

The flag variety of pointed lines in projective 3-space is associated to the Dynkin diagram `\dynkin[parabolic=3]{A}{3}`.

The flag variety of pointed lines in projective 3-space is associated to the Dynkin diagram  $\times \rightarrow \bullet$ .

Table 8: The Hermitian symmetric spaces

$A_n$	$\bullet \cdots \bullet \times \cdots \bullet$	Grassmannian of $k$ -planes in $\mathbb{C}^{n+1}$
$B_n$	$\times \cdots \bullet \rightarrow \bullet$	$(2n-1)$ -dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n+1}$
$C_n$	$\bullet \cdots \bullet \leftarrow \times$	space of Lagrangian $n$ -planes in $\mathbb{C}^{2n}$
$D_n$	$\times \cdots \bullet \begin{array}{l} \nearrow \bullet \\ \searrow \bullet \end{array}$	$(2n-2)$ -dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n}$
$D_n$	$\bullet \cdots \bullet \begin{array}{l} \nearrow \bullet \\ \searrow \times \end{array}$	one component of the variety of maximal dimension null subspaces of $\mathbb{C}^{2n}$
$D_n$	$\bullet \cdots \bullet \begin{array}{l} \nearrow \times \\ \searrow \bullet \end{array}$	the other component
$E_6$	$\times \cdots \bullet \begin{array}{l} \uparrow \bullet \\ \downarrow \bullet \end{array}$	complexified octave projective plane
$E_6$	$\bullet \cdots \bullet \begin{array}{l} \uparrow \bullet \\ \downarrow \times \end{array}$	its dual plane
$E_7$	$\bullet \cdots \bullet \begin{array}{l} \uparrow \bullet \\ \downarrow \times \end{array}$	the space of null octave 3-planes in octave 6-space

```

\NewDocumentCommand\HSS{mommm}
{#1&\IfNoValueTF{#2}{\dynkin{#3}{#4}}{\dynkin[parabolic=#2]{#3}{#4}}&#5\\}
\renewcommand*{\arraystretch}{1.5}
\begin{longtable}
{>\columncolor[gray]{.9}}>$1<$>\columncolor[gray]{.9}}>$1<$>\columncolor[gray]{.9}}1}
\caption{The Hermitian symmetric spaces}\endfirsthead
\caption{\dots continued}\\ \endhead
\caption{continued \dots}\\ \endfoot
\endlastfoot
\HSS{A_n}{A}{**.*x*.*}{Grassmannian of $k$-planes in $\mathbb{C}^{n+1}$}
\HSS{B_n}[1]{B}{-}{$(2n-1)$-dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n+1}$}
\HSS{C_n}[16]{C}{-}{space of Lagrangian $n$-planes in $\mathbb{C}^{2n}$}
\HSS{D_n}[1]{D}{-}{$(2n-2)$-dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n}$}
\HSS{D_n}[32]{D}{-}{one component of the variety of maximal dimension null subspaces of $\mathbb{C}^{2n}$}
\HSS{D_n}[16]{D}{-}{the other component}
\HSS{E_6}[1]{E}{6}{complexified octave projective plane}
\HSS{E_6}[32]{E}{6}{its dual plane}
\HSS{E_7}[64]{E}{7}{the space of null octave 3-planes in octave 6-space}
\end{longtable}

```

Folded parabolics look bad (zoom in on a root)

```
\dynkin[fold,parabolic=3]{C}{2}
\dynkin[fold,parabolic=3]{G}{2}
```



Folded parabolics: you can try using thicker crosses

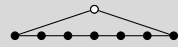
```
\dynkin[fold,x/.style={very thick,line cap=round},parabolic=3]{C}{2}
\dynkin[fold,x/.style={ultra thick,line cap=round},parabolic=3]{G}{2}
```



## 18. EXTENDED DYNKIN DIAGRAMS

Extended Dynkin diagrams

```
\dynkin[extended]{A}{7}
```



The extended Dynkin diagrams are also described in the notation of Kac [15] p. 55 as affine untwisted Dynkin diagrams: we extend `\dynkin{A}{7}` to become `\dynkin{A}[1]{7}`:

Extended Dynkin diagrams

```
\dynkin{A}[1]{7}
```

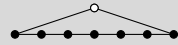


Table 9: The Dynkin diagrams of the extended simple root systems

$A_1^1$		<code>\dynkin[extended]{A}{1}</code>
$A_n^1$		<code>\dynkin[extended]{A}{}</code>
$B_n^1$		<code>\dynkin[extended]{B}{}</code>

continued ...

Table 9: ...continued

$C_n^1$		<code>\dynkin[extended]{C}{}</code>
$D_n^1$		<code>\dynkin[extended]{D}{}</code>
$E_6^1$		<code>\dynkin[extended]{E}{6}</code>
$E_7^1$		<code>\dynkin[extended]{E}{7}</code>
$E_8^1$		<code>\dynkin[extended]{E}{8}</code>
$F_4^1$		<code>\dynkin[extended]{F}{4}</code>
$G_2^1$		<code>\dynkin[extended]{G}{2}</code>

## 19. AFFINE TWISTED AND UNTWISTED DYNKIN DIAGRAMS

The affine Dynkin diagrams are described in the notation of Kac [15] p. 55:

Affine Dynkin diagrams

$\backslash(A^{(1)})_7=\backslash\mathrm{dynkin}\{A\}[1]\{7\}, \backslash$   
 $E^{(2)}_6=\backslash\mathrm{dynkin}\{E\}[2]\{6\}, \backslash$   
 $D^{(3)}_4=\backslash\mathrm{dynkin}\{D\}[3]\{4\}\backslash$

$$A_7^{(1)} = \text{diagram}, E_6^{(2)} = \text{diagram}, D_4^{(3)} = \text{diagram}$$

Table 10: The affine Dynkin diagrams

$A_1^1$		<code>\dynkin{A}[1]\{1\}</code>
$A_n^1$		<code>\dynkin{A}[1]\{n\}</code>
$B_n^1$		<code>\dynkin{B}[1]\{n\}</code>
$C_n^1$		<code>\dynkin{C}[1]\{n\}</code>
$D_n^1$		<code>\dynkin{D}[1]\{n\}</code>
$E_6^1$		<code>\dynkin{E}[1]\{6\}</code>
$E_7^1$		<code>\dynkin{E}[1]\{7\}</code>
$E_8^1$		<code>\dynkin{E}[1]\{8\}</code>
$F_4^1$		<code>\dynkin{F}[1]\{4\}</code>

continued ...

Table 10: ... continued

$G_2^1$		<code>\dynkin{G}{1}{2}</code>
$A_2^2$		<code>\dynkin{A}{2}{2}</code>
$A_{ev}^2$		<code>\dynkin{A}{2}{even}</code>
$A_{od}^2$		<code>\dynkin{A}{2}{odd}</code>
$D_n^2$		<code>\dynkin{D}{2}{}</code>
$E_6^2$		<code>\dynkin{E}{2}{6}</code>
$D_4^3$		<code>\dynkin{D}{3}{4}</code>

Table 11: Some more affine Dynkin diagrams

$A_4^2$		<code>\dynkin{A}{2}{4}</code>
$A_5^2$		<code>\dynkin{A}{2}{5}</code>
$A_6^2$		<code>\dynkin{A}{2}{6}</code>
$A_7^2$		<code>\dynkin{A}{2}{7}</code>
$A_8^2$		<code>\dynkin{A}{2}{8}</code>
$D_3^2$		<code>\dynkin{D}{2}{3}</code>
$D_4^2$		<code>\dynkin{D}{2}{4}</code>
$D_5^2$		<code>\dynkin{D}{2}{5}</code>
$D_6^2$		<code>\dynkin{D}{2}{6}</code>
$D_7^2$		<code>\dynkin{D}{2}{7}</code>
$D_8^2$		<code>\dynkin{D}{2}{8}</code>
$D_4^3$		<code>\dynkin{D}{3}{4}</code>
$E_6^2$		<code>\dynkin{E}{2}{6}</code>

## 20. EXTENDED COXETER DIAGRAMS

Extended and Coxeter options together

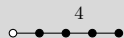
`\dynkin[extended,Coxeter]{F}{4}`

Table 12: The extended (affine) Coxeter diagrams

$A_n$		<code>\dynkin[extended,Coxeter]{A}{}</code>
$B_n$		<code>\dynkin[extended,Coxeter]{B}{}</code>
$C_n$		<code>\dynkin[extended,Coxeter]{C}{}</code>
$D_n$		<code>\dynkin[extended,Coxeter]{D}{}</code>
$E_6$		<code>\dynkin[extended,Coxeter]{E}{6}</code>
$E_7$		<code>\dynkin[extended,Coxeter]{E}{7}</code>
$E_8$		<code>\dynkin[extended,Coxeter]{E}{8}</code>
$F_4$		<code>\dynkin[extended,Coxeter]{F}{4}</code>
$G_2$		<code>\dynkin[extended,Coxeter]{G}{2}</code>
$H_3$		<code>\dynkin[extended,Coxeter]{H}{3}</code>
$H_4$		<code>\dynkin[extended,Coxeter]{H}{4}</code>
$I_1$		<code>\dynkin[extended,Coxeter]{I}{1}</code>

## 21. KAC STYLE

We include a style called `Kac` which tries to imitate the style of [15].

Kac style

`\dynkin[Kac]{F}{4}`

$\circ \text{---} \circ \Rightarrow \circ \text{---} \circ$

Table 13: The Dynkin diagrams of the simple root systems in Kac style

$A_n$	$\circ \text{---} \circ \text{---} \cdots \text{---} \circ \text{---} \circ$	<code>\dynkin{A}{}</code>
$B_n$	$\circ \text{---} \circ \text{---} \cdots \text{---} \circ \text{---} \circ \Rightarrow \circ$	<code>\dynkin{B}{}</code>
$C_n$	$\circ \text{---} \circ \text{---} \cdots \text{---} \circ \text{---} \circ \Leftarrow \circ$	<code>\dynkin{C}{}</code>
$D_n$	$\circ \text{---} \circ \text{---} \cdots \text{---} \circ \text{---} \circ \begin{array}{l} \nearrow \circ \\ \searrow \circ \end{array}$	<code>\dynkin{D}{}</code>

continued ...

Table 13: ...continued

$E_6$		<code>\dynkin{E}{6}</code>
$E_7$		<code>\dynkin{E}{7}</code>
$E_8$		<code>\dynkin{E}{8}</code>
$F_4$	$\circ - \circ \Rightarrow \circ - \circ$	<code>\dynkin{F}{4}</code>
$G_2$	$\circ \Rrightarrow \circ$	<code>\dynkin{G}{2}</code>

Table 14: The Dynkin diagrams of the extended simple root systems in Kac style

$A_1^1$	$\Leftrightarrow \circ$	<code>\dynkin[extended]{A}{1}</code>
$A_n^1$		<code>\dynkin[extended]{A}{}</code>
$B_n^1$		<code>\dynkin[extended]{B}{}</code>
$C_n^1$	$\circ \Rightarrow \circ - \circ - \dots - \circ - \circ \Leftarrow \circ$	<code>\dynkin[extended]{C}{}</code>
$D_n^1$		<code>\dynkin[extended]{D}{}</code>
$E_6^1$		<code>\dynkin[extended]{E}{6}</code>
$E_7^1$		<code>\dynkin[extended]{E}{7}</code>
$E_8^1$		<code>\dynkin[extended]{E}{8}</code>
$F_4^1$	$\circ - \circ - \circ \Rightarrow \circ - \circ$	<code>\dynkin[extended]{F}{4}</code>
$G_2^1$	$\circ - \circ \Rrightarrow \circ$	<code>\dynkin[extended]{G}{2}</code>

Table 15: The Dynkin diagrams of the twisted simple root systems in Kac style

$A_2^2$	$\circ \Leftarrow \circ$	<code>\dynkin{A}{2}{2}</code>
$A_{ev}^2$	$\circ \Leftarrow \circ - \circ - \circ - \cdots - \circ - \circ \Leftarrow \circ$	<code>\dynkin{A}{2}{even}</code>
$A_{od}^2$	$\begin{array}{c} \circ \\ \diagup \\ \circ - \circ - \circ - \cdots - \circ - \circ \Leftarrow \circ \\ \diagdown \\ \circ \end{array}$	<code>\dynkin{A}{2}{odd}</code>
$D_n^2$	$\circ \Leftarrow \circ - \circ - \cdots - \circ - \circ \Rightarrow \circ$	<code>\dynkin{D}{2}{}</code>
$E_6^2$	$\circ - \circ - \circ \Leftarrow \circ - \circ$	<code>\dynkin{E}{2}{6}</code>
$D_4^3$	$\circ - \circ \Leftarrow \circ$	<code>\dynkin{D}{3}{4}</code>

## 22. CEREF STYLE

We include a style called `ceref` which paints oblong root markers with shadows. The word “ceref” is an old form of the word “serif”.

Ceref style

`\dynkin[ceref]{F}{4}`

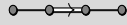


Table 16: The Dynkin diagrams of the simple root systems in ceref style

$A_n$	$\bullet - \bullet - \cdots - \bullet$	<code>\dynkin{A}{}</code>
$B_n$	$\bullet - \bullet - \cdots - \bullet \Rightarrow \bullet$	<code>\dynkin{B}{}</code>
$C_n$	$\bullet - \bullet - \cdots - \bullet \Leftarrow \bullet$	<code>\dynkin{C}{}</code>
$D_n$	$\bullet - \bullet - \cdots - \bullet \begin{array}{l} \diagup \bullet \\ \diagdown \bullet \end{array}$	<code>\dynkin{D}{}</code>
$E_6$	$\begin{array}{c} \bullet \\   \\ \bullet - \bullet - \bullet - \bullet - \bullet \end{array}$	<code>\dynkin{E}{6}</code>
$E_7$	$\begin{array}{c} \bullet \\   \\ \bullet - \bullet - \bullet - \bullet - \bullet - \bullet \end{array}$	<code>\dynkin{E}{7}</code>
$E_8$	$\begin{array}{c} \bullet \\   \\ \bullet - \bullet - \bullet - \bullet - \bullet - \bullet - \bullet \end{array}$	<code>\dynkin{E}{8}</code>
$F_4$	$\bullet - \bullet \Rightarrow \bullet - \bullet$	<code>\dynkin{F}{4}</code>
$G_2$	$\bullet \Rightarrow \bullet$	<code>\dynkin{G}{2}</code>

Table 17: The Dynkin diagrams of the extended simple root systems in ceref style

$A_1^1$		<code>\dynkin[extended]{A}{1}</code>
$A_n^1$		<code>\dynkin[extended]{A}{}</code>
$B_n^1$		<code>\dynkin[extended]{B}{}</code>
$C_n^1$		<code>\dynkin[extended]{C}{}</code>
$D_n^1$		<code>\dynkin[extended]{D}{}</code>
$E_6^1$		<code>\dynkin[extended]{E}{6}</code>
$E_7^1$		<code>\dynkin[extended]{E}{7}</code>
$E_8^1$		<code>\dynkin[extended]{E}{8}</code>
$F_4^1$		<code>\dynkin[extended]{F}{4}</code>
$G_2^1$		<code>\dynkin[extended]{G}{2}</code>

Table 18: The Dynkin diagrams of the twisted simple root systems in ceref style

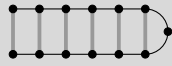
$A_2^2$		<code>\dynkin{A}{2}{2}</code>
$A_{ev}^2$		<code>\dynkin{A}{2}{even}</code>
$A_{od}^2$		<code>\dynkin{A}{2}{odd}</code>
$D_n^2$		<code>\dynkin{D}{2}{}</code>
$E_6^2$		<code>\dynkin{E}{2}{6}</code>
$D_4^3$		<code>\dynkin{D}{3}{4}</code>

### 23. MORE ON FOLDED DYNKIN DIAGRAMS

The Dynkin diagrams package has limited support for folding Dynkin diagrams.

#### Folding

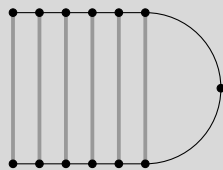
`\dynkin[fold]{A}{13}`





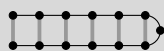
## Big fold radius

```
\dynkin[fold,fold radius=1cm]{A}{13}
```



## Small fold radius

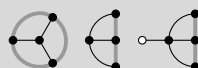
```
\dynkin[fold,fold radius=.2cm]{A}{13}
```



Some Dynkin diagrams have multiple foldings, which we attempt to distinguish (not entirely successfully) by their *ply*: the maximum number of roots folded together. Most diagrams can only allow a 2-ply folding, so fold is a synonym for ply=2.

## 3-ply

```
\dynkin[ply=3]{D}{4}
\dynkin[ply=3,fold right]{D}{4}
\dynkin[ply=3]{D}[1]{4}
```



## 4-ply

```
\dynkin[ply=4]{D}[1]{4}
```



The  $D_\ell^{(1)}$  diagrams can be folded on their left end and separately on their right end:

Left, right and both

```
\dynkin{D}[1]{ } \
\dynkin[fold left]{D}[1]{ } \
\dynkin[fold right]{D}[1]{ } \
\dynkin[fold]{D}[1]{ }
```



We have to be careful about the 4-ply foldings of  $D_{2\ell}^{(1)}$ , for which we can have two different patterns, so by default, the package only draws as much as it can without distinguishing the two:

Default  $D_{2\ell}^{(1)}$  and the two ways to finish it

```
\dynkin[ply=4]{D}[1]{****.*****.*****}%
\
\begin{dynkinDiagram}[ply=4]{D}[1]{****.*****.*****}%
  \dynkinFold[bend right=90]{1}{13}%
  \dynkinFold[bend right=90]{0}{14}%
\end{dynkinDiagram} \
\begin{dynkinDiagram}[ply=4]{D}[1]{****.*****.*****}%
  \dynkinFold{0}{1}%
  \dynkinFold{1}{13}%
  \dynkinFold{13}{14}%
\end{dynkinDiagram}
```

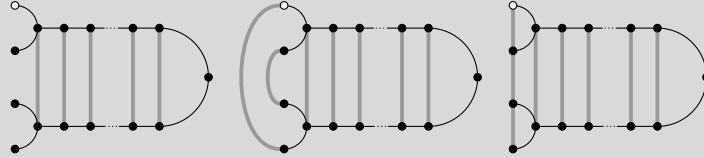


Table 19: Some foldings of Dynkin diagrams. For these diagrams, we want to compare a folding diagram with the diagram that results when we fold it, so it looks best to set `fold radius` and `edge length` to equal lengths.

$A_3$		<code>\dynkin[fold]{A}[0]{3}</code>
$C_2$		<code>\dynkin{C}[0]{2}</code>

continued ...

Table 19: ...continued

$A_{2\ell-1}$		<code>\dynkin[fold]{A}{**.****.**}</code>
$C_\ell$		<code>\dynkin{C}{}</code>
$B_3$		<code>\dynkin[fold]{B}{0}{3}</code>
$G_2$		<code>\dynkin[reverse arrows]{G}{0}{2}</code>
$D_4$		<code>\dynkin[ply=3,fold right]{D}{4}</code>
$G_2$		<code>\dynkin{G}{2}</code>
$D_{\ell+1}$		<code>\dynkin[fold]{D}{}</code>
$B_\ell$		<code>\dynkin{B}{}</code>
$E_6$		<code>\dynkin[fold]{E}{0}{6}</code>
$F_4$		<code>\dynkin[reverse arrows]{F}{0}{4}</code>
$A_3^1$		<code>\dynkin[ply=4]{A}{1}{3}</code>
$A_1^1$		<code>\dynkin{A}{1}{1}</code>
$A_{2\ell-1}^1$		<code>\dynkin[fold]{A}{1}{**.****.**}</code>
$C_\ell^1$		<code>\dynkin{C}{1}{}</code>
$B_3^1$		<code>\dynkin[ply=3]{B}{1}{3}</code>
$A_2^2$		<code>\dynkin{A}{2}{2}</code>
$B_3^1$		<code>\dynkin[ply=2]{B}{1}{3}</code>
$G_2^1$		<code>\dynkin{G}{1}{2}</code>
$B_\ell^1$		<code>\dynkin[fold]{B}{1}{}</code>
$D_\ell^2$		<code>\dynkin{D}{2}{}</code>
$D_4^1$		<code>\dynkin[ply=3]{D}{1}{4}</code>
$B_3^1$		<code>\dynkin{B}{1}{3}</code>

continued ...

Table 19: ...continued

$D_4^1$		<code>\dynkin[ply=3]{D}[1]{4}</code>
$G_2^1$		<code>\dynkin{G}[1]{2}</code>
$D_{\ell+1}^1$		<code>\dynkin[fold]{D}[1]{}</code>
$D_\ell^2$		<code>\dynkin{D}[2]{}</code>
$D_{\ell+1}^1$		<code>\dynkin[fold right]{D}[1]{}</code>
$B_\ell^1$		<code>\dynkin{B}[1]{}</code>
$D_{2\ell}^1$		<pre> \begin{dynkinDiagram}[ply=4]{D}[1]% {****.*****.*****} \dynkinFold{0}{1} \dynkinFold{1}{13} \dynkinFold{13}{14} \end{dynkinDiagram} </pre>
$A_{\text{odd}}^2$		<code>\dynkin{A}[2]{odd}</code>
$D_{2\ell}^1$		<pre> \begin{dynkinDiagram}[ply=4]{D}[1]% {****.*****.*****} \dynkinFold[bend right=90]{1}{13} \dynkinFold[bend right=90]{0}{14} \end{dynkinDiagram} </pre>
$A_{\text{even}}^2$		<code>\dynkin{A}[2]{even}</code>
$E_6^1$		<code>\dynkin[fold]{E}[1]{6}</code>
$F_4^1$		<code>\dynkin[reverse arrows]{F}[1]{4}</code>
$E_6^1$		<code>\dynkin[ply=3]{E}[1]{6}</code>
$D_4^3$		<code>\dynkin{D}[3]{4}</code>
$E_7^1$		<code>\dynkin[fold]{E}[1]{7}</code>
$E_6^2$		<code>\dynkin{E}[2]{6}</code>

continued ...

Table 19: ...continued

$F_4^1$		<code>\dynkin[fold]{F}{1}{4}</code>
$G_2^1$		<code>\dynkin{G}{1}{2}</code>
$A_{\text{odd}}^2$		<code>\dynkin[odd, fold]{A}{2}{****,***}</code>
$A_{\text{even}}^2$		<code>\dynkin{A}{2}{even}</code>
$D_3^2$		<code>\dynkin[fold]{D}{2}{3}</code>
$A_2^2$		<code>\dynkin{A}{2}{2}</code>

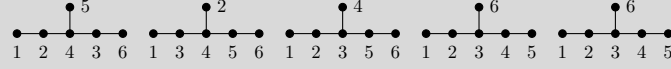
Table 20: Frobenius fixed point subgroups of finite simple groups of Lie type [4] p. 15

$A_{\ell \geq 1}$		<code>\dynkin{A}{}</code>
${}^2A_{\ell \geq 2}$		<code>\dynkin[fold]{A}{}</code>
$B_{\ell \geq 2}$		<code>\dynkin{B}{}</code>
${}^2B_2$		<code>\dynkin[fold]{B}{2}</code>
$C_{\ell \geq 3}$		<code>\dynkin{C}{}</code>
$D_{\ell \geq 4}$		<code>\dynkin{D}{}</code>
${}^2D_{\ell \geq 4}$		<code>\dynkin[fold]{D}{}</code>
${}^3D_4$		<code>\dynkin[poly=3]{D}{4}</code>
$E_6$		<code>\dynkin{E}{6}</code>
${}^2E_6$		<code>\dynkin[fold]{E}{6}</code>
$E_7$		<code>\dynkin{E}{7}</code>
$E_8$		<code>\dynkin{E}{8}</code>
$F_4$		<code>\dynkin{F}{4}</code>
${}^2F_4$		<code>\dynkin[fold]{F}{4}</code>
$G_2$		<code>\dynkin{G}{2}</code>
${}^2G_2$		<code>\dynkin[fold]{G}{2}</code>

## 24. ROOT ORDERING

Root ordering

```
\dynkin[label,ordering=Adams]{E}{6}
\dynkin[label,ordering=Bourbaki]{E}{6}
\dynkin[label,ordering=Carter]{E}{6}
\dynkin[label,ordering=Dynkin]{E}{6}
\dynkin[label,ordering=Kac]{E}{6}
```

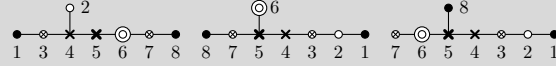


Default is Bourbaki. Sources are Adams [1] p. 56–57, Bourbaki [3] p. pp. 265–290 plates I–IX, Carter [5] p. 540–609, Dynkin [8], Kac [15] p. 43.

	Adams	Bourbaki	Carter	Dynkin	Kac
$E_6$					
$E_7$					
$E_8$					
$F_4$					
$G_2$					

The marks are set down in order according to the current root ordering:

```
\dynkin[label]{E}{*otxXOt*}
\dynkin[label,ordering=Carter]{E}{*otxXOt*}
\dynkin[label,ordering=Kac]{E}{*otxXOt*}
```

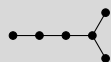


## 25. CONNECTING DYNKIN DIAGRAMS

We can make some sophisticated folded diagrams by drawing multiple diagrams, each with a name:

Name a diagram

```
\dynkin[name=Bob]{D}{6}
```



We can then connect the two with folding edges:

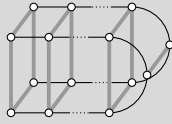
Connect diagrams

```
\begin{dynkinDiagram}[name=upper]{A}{3}
  \node (current) at ($(\text{upper root 1})+(0,-.3\text{cm})$) {};
  \dynkin[at=(current),name=lower]{A}{3}
  \begin{scope}[on background layer]
    \foreach \i in {1,...,3}%
    {%
      \draw[/Dynkin diagram/fold style]
        ($(\text{upper root \i})$)
        -- ($(\text{lower root \i})$);%
    }%
  \end{scope}
\end{dynkinDiagram}
```



The following diagrams arise in the Satake diagrams of the pseudo-Riemannian symmetric spaces [2].

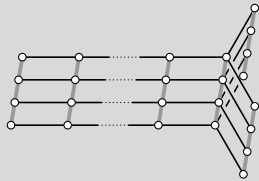
```
\pgfkeys{/Dynkin diagram,edge length=.5cm,fold radius=.5cm}
\begin{tikzpicture}
  \dynkin[name=1]{A}{IIIb}
  \node (a) at (-.3,-.4){};
  \dynkin[name=2,at=(a)]{A}{IIIb}
  \begin{scope}[on background layer]
    \foreach \i in {1,...,7}%
    {%
      \draw[/Dynkin diagram/fold style]
        ($(\text{1 root \i})$)
        --
        ($(\text{2 root \i})$);%
    }%
  \end{scope}
\end{tikzpicture}
```



```

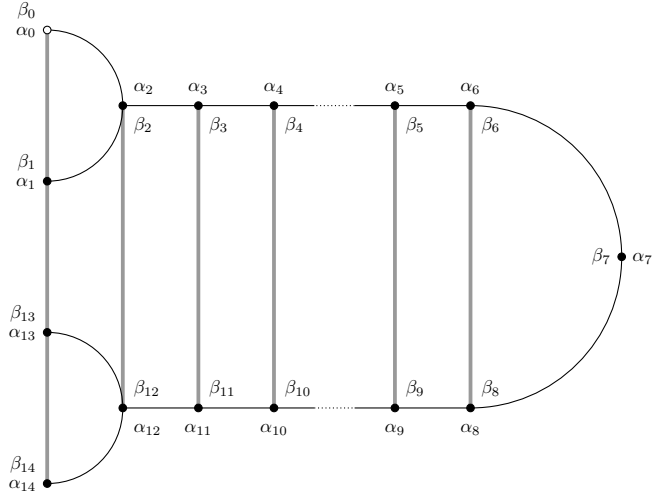
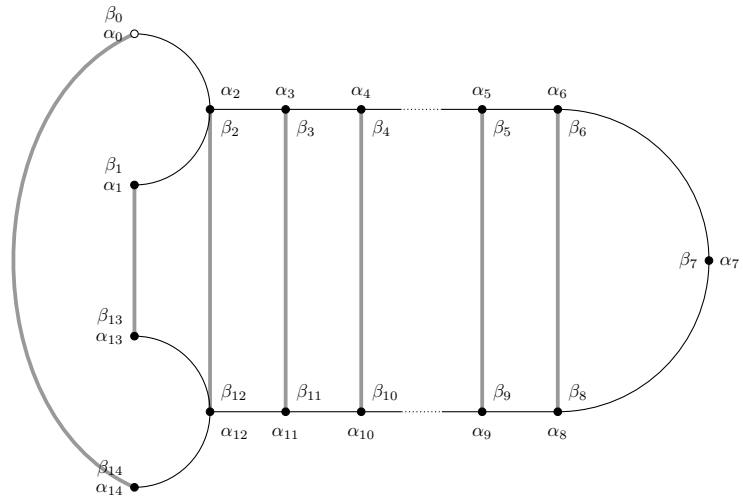
\pgfkeys{/Dynkin diagram,
edge length=.75cm,
edge/.style={draw=example-color,double=black,very thick}}
\begin{tikzpicture}
  \foreach \d in {1,...,4}
  {
    \node (current) at ($(\d*.05,\d*.3)$){};
    \dynkin[name=\d,at=(current)]{D}{oo.oooo}
  }
  \begin{scope}[on background layer]
    \foreach \i in {1,...,6}%
    {%
      \draw[/Dynkin diagram/fold style] ($ (1 root \i)$) -- ($ (2
root \i)$);%
      \draw[/Dynkin diagram/fold style] ($ (2 root \i)$) -- ($ (3
root \i)$);%
      \draw[/Dynkin diagram/fold style] ($ (3 root \i)$) -- ($ (4
root \i)$);%
    }%
  \end{scope}
\end{tikzpicture}

```





## 26. OTHER EXAMPLES

 ${}^1D_4$  4-ply tied straight: ${}^1D_4$  4-ply tied bending:

```

\tikzset{/Dynkin diagram,edge length=1cm,fold radius=1cm}
\tikzset{/Dynkin diagram,label macro/.code={\alpha_{#1}},label macro*/.code={\beta_{#1}}}
\({}^1D_4\) 4-ply tied straight:
\begin{dynkinDiagram}[ply=4]{D}[1]%
{****,****,****}
\dynkinFold{0}{1}
\dynkinFold{1}{13}
\dynkinFold{13}{14}
\dynkinLabelRoots{0,...,14}
\dynkinLabelRoots*{0,...,14}
\end{dynkinDiagram}
\({}^1D_4\) 4-ply tied bending:
\begin{dynkinDiagram}[ply=4]{D}[1]%
{****,****,****}
\dynkinFold{1}{13}
\dynkinFold[bend right=65]{0}{14}
\dynkinLabelRoots{0,...,14}

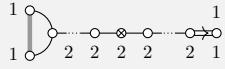
```

```
\dynkinLabelRoots*{0,...,14}
\end{dynkinDiagram}
```

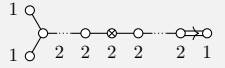
Below we draw the Vogan diagrams of some affine Lie superalgebras [21, 20].

$\mathfrak{sl}(2m|2n)^{(2)}$

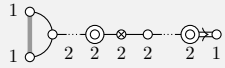
```
\begin{dynkinDiagram}[ply=2,label]{B}[1]{oo.oto.oo}
\dynkinLabelRoot*{7}{1}
\end{dynkinDiagram}
```



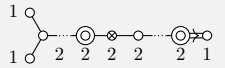
```
\dynkin[label]{B}[1]{oo.oto.oo}
```



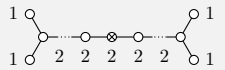
```
\dynkin[ply=2,label]{B}[1]{oo.Oto.Oo}
```



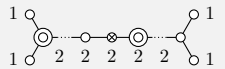
```
\dynkin[label]{B}[1]{oo.Oto.Oo}
```

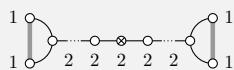


```
\dynkin[label]{D}[1]{oo.oto.ooo}
```

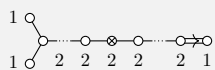


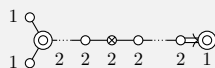
```
\dynkin[label]{D}[1]{oO.otO.ooo}
```

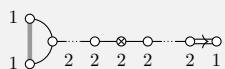


$$\backslash\text{dynkin}[\text{label},\text{fold}]\{\text{D}\}[1]\{\text{oo.oto.ooo}\}$$


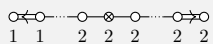
$$\mathfrak{sl}(2m+1|2n)^2$$

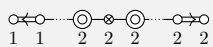
$$\backslash\text{dynkin}[\text{label}]\{\text{B}\}[1]\{\text{oo.oto.oo}\}$$


$$\backslash\text{dynkin}[\text{label}]\{\text{B}\}[1]\{\text{oO.oto.oO}\}$$


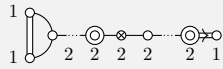
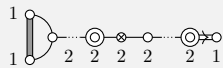
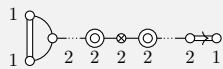
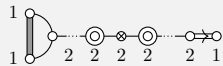
$$\backslash\text{dynkin}[\text{label},\text{fold}]\{\text{B}\}[1]\{\text{oo.oto.oo}\}$$


$$\mathfrak{sl}(2m+1|2n+1)^2$$

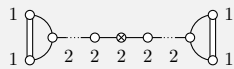
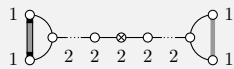
$$\backslash\text{dynkin}[\text{label}]\{\text{D}\}[2]\{\text{o.oto.oo}\}$$


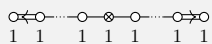
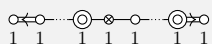
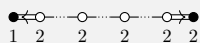
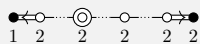
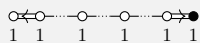
$$\backslash\text{dynkin}[\text{label}]\{\text{D}\}[2]\{\text{o.OtO.oo}\}$$


$$\mathfrak{sl}(2|2n+1)^{(2)}$$

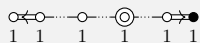
$$\backslash \text{dynkin}[ply=2, \text{label}, \text{double edges}]\{\text{B}\}[1]\{\text{oo.Oto.Oo}\}$$

$$\backslash \text{dynkin}[ply=2, \text{label}, \text{double fold}]\{\text{B}\}[1]\{\text{oo.Oto.Oo}\}$$

$$\backslash\mathrm{dynkin}[ply=2,label,double\ edges]\{B\}[1]\{oo.OtO.oo\}$$

$$\backslash\mathrm{dynkin}[ply=2,label,double\ fold]\{B\}[1]\{oo.OtO.oo\}$$


$$\mathfrak{sl}(2|2n)^{(2)}$$

$$\backslash \text{dynkin}[\text{ply}=2, \text{label}, \text{double edges}]\{\text{D}\}[1]\{\text{oo.oto.ooo}\}$$

$$\backslash\mathrm{dynkin}[ply=2,label,double\ fold\ left]\{D\}[1]\{oo.oto.ooo\}$$


$\mathfrak{osp}(2m|2n)^{(2)}$ 
`\dynkin[label,label macro/.code={1}]{D}[2]{o.oto.oo}`

`\dynkin[label,label macro/.code={1}]{D}[2]{o.Oto.Oo}`

 $\mathfrak{osp}(2|2n)^{(2)}$ 
`\dynkin[label,label macro/.code=\lablIt{#1},  
affine mark=*]  
{D}[2]{o.o.o.o*}`

`\dynkin[label,label macro/.code=\lablIt{#1},  
affine mark=*]  
{D}[2]{o.O.o.o*}`

 $\mathfrak{sl}(1|2n+1)^4$ 
`\dynkin[label,label macro/.code={1}]{D}[2]{o.o.o.o*}`


```
\dynkin[label,label macro/.code={1}]{D}[2]{o.o.O.o*}
```


 $A^1$ 

```
\begin{tikzpicture}
  \dynkin[name=upper]{A}{oo.t.oo}
  \node (Dynkin current) at (upper root 1){};
  \dynkinSouth
  \dynkin[at=(Dynkin
current),name=lower]{A}{oo.t.oo}
  \begin{scope}[on background layer]
    \foreach \i in {1,...,5}{
      \draw[/Dynkin diagram/fold style]
        ($(\text{upper root \i})$) -- ($(\text{lower root \i})$);
    }
  \end{scope}
\end{tikzpicture}
```



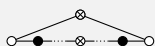
```
\dynkin[fold]{A}[1]{oo.t.oooo.t.oo}
```

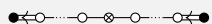
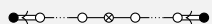
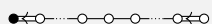
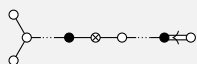
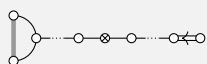
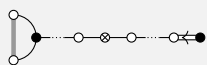


```
\dynkin[fold,affine mark=t]{A}[1]{oo.o.oootoo.o.oo}
```



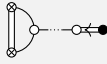
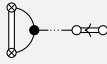
```
\dynkin[affine mark=t]{A}[1]{o*.t.*o}
```



$B^1$ `\dynkin[affine mark=*]{A}[2]{o.oto.o*}``\dynkin[affine mark=*]{A}[2]{o.oto.o*}``\dynkin[affine mark=*]{A}[2]{o.ooo.oo}``\dynkin[odd]{A}[2]{oo.*to.*o}``\dynkin[odd,fold]{A}[2]{oo.oto.oo}``\dynkin[odd,fold]{A}[2]{o*.oto.o*}` $D^1$ `\dynkin{D}{otoo}`

$\backslash\text{dynkin}\{D\}\{\text{ot}*\text{o}\}$ 

 $\backslash\text{dynkin}[\text{fold}]\{D\}\{\text{otoo}\}$ 

 $C^1$ 
 $\backslash\text{dynkin}[\text{double edges},\text{fold},\text{affine mark}=\text{t},\text{odd}]\{A\}[2]\{\text{to.o}*\}$ 

 $\backslash\text{dynkin}[\text{double edges},\text{fold},\text{affine mark}=\text{t},\text{odd}]\{A\}[2]\{\text{t}*.oo\}$ 

 $F^1$ 
 $\backslash\text{begin}\{\text{dynkinDiagram}\}\{A\}\{\text{oto}*\}\%$   
 $\quad\backslash\text{dynkinQuadrupleEdge}\{1\}\{2\}\%$   
 $\quad\backslash\text{dynkinTripleEdge}\{4\}\{3\}\%$   
 $\backslash\text{end}\{\text{dynkinDiagram}\}\%$ 

 $\backslash\text{begin}\{\text{dynkinDiagram}\}\{A\}\{*\text{too}\}\%$   
 $\quad\backslash\text{dynkinQuadrupleEdge}\{1\}\{2\}\%$   
 $\quad\backslash\text{dynkinTripleEdge}\{4\}\{3\}\%$   
 $\backslash\text{end}\{\text{dynkinDiagram}\}\%$ 






## 27. EXAMPLE: THE COMPLEX SIMPLE LIE ALGEBRAS

$\mathfrak{g}$	Diagram	Weights	Roots	Simple roots
$A_n$		$\frac{1}{n+1}\mathbb{Z}^{n+1} / \langle \sum e_j \rangle$	$e_i - e_j$	$e_i - e_{i+1}$
$B_n$		$\frac{1}{2}\mathbb{Z}^n$	$\pm e_i, \pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, e_n$
$C_n$		$\mathbb{Z}^n$	$\pm 2e_i, \pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, 2e_n$
$D_n$		$\frac{1}{2}\mathbb{Z}^n$	$\pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, \quad i \leq n-1$ $e_{n-1} + e_n$

$\mathfrak{g}$	Diagram	Weights	Roots	Simple roots
$E_8$		$\frac{1}{2}\mathbb{Z}^8$	$\pm 2e_i \pm 2e_j, \quad i \neq j,$ $\sum_i (-1)^{m_i} e_i, \quad \sum m_i \text{ even}$	$2e_1 - 2e_2,$ $2e_2 - 2e_3,$ $2e_3 - 2e_4,$ $2e_4 - 2e_5,$ $2e_5 - 2e_6,$ $2e_6 + 2e_7,$ $-\sum e_j,$ $2e_6 - 2e_7$
$E_7$		$\frac{1}{2}\mathbb{Z}^8 / \langle e_1 - e_2 \rangle$	quotient of $E_8$	quotient of $E_8$
$E_6$		$\frac{1}{3}\mathbb{Z}^8 / \langle e_1 - e_2, e_2 - e_3 \rangle$	quotient of $E_8$	quotient of $E_8$
$F_4$		$\mathbb{Z}^4$	$\pm 2e_i,$ $\pm 2e_i \pm 2e_j, \quad i \neq j,$ $\pm e_1 \pm e_2 \pm e_3 \pm e_4$	$2e_2 - 2e_3,$ $2e_3 - 2e_4,$ $2e_4,$ $e_1 - e_2 - e_3 - e_4$
$G_2$		$\mathbb{Z}^3 / \langle \sum e_j \rangle$	$\pm(1, -1, 0),$ $\pm(-1, 0, 1),$ $\pm(0, -1, 1),$ $\pm(2, -1, -1),$ $\pm(1, -2, 1),$ $\pm(-1, -1, 2)$	$(-1, 0, 1),$ $(2, -1, -1)$

```

\NewDocumentEnvironment{bunch}{}%
{\renewcommand*{\arraystretch}{1}\begin{array}{@{}l@{}}\midrule{\midrule\end{array}}
\small
\NewDocumentCommand\nc{mm}{\newcolumntype{#1}{\columncolor[gray]{.9}}>{\$}m{#2cm}<{\$}}
\nc{G}{.3}\nc{D}{2.1}\nc{W}{3}\nc{R}{3.7}\nc{S}{3}
\NewDocumentCommand\LieG{}{\mathfrak{g}}
\NewDocumentCommand\W{om}{\ensuremath{\mathbb{Z}}^{\#2}\IfValueT{#1}{/\left<#1\right>}}
\renewcommand*{\arraystretch}{1.5}
\NewDocumentCommand\quo{}{\text{quotient of } E_8}
\begin{longtable}{@{}GDWRS@{}}
\LieG&\text{Diagram}&\text{Weights}&\text{Roots}&\text{Simple roots}\midrule\endfirsthead
\LieG&\text{Diagram}&\text{Weights}&\text{Roots}&\text{Simple roots}\midrule\endhead
A_n&\dynkin{A}{n}&\frac{1}{n+1}\W{\sum e_j}{n+1}&e_i-e_j&e_i-e_{i+1}\\
B_n&\dynkin{B}{n}&\frac{1}{2}\W{n}&\pm e_i, \pm e_i \pm e_j, i \neq j&e_i-e_{i+1}, e_n\\
C_n&\dynkin{C}{n}&\frac{1}{2}\W{n}&\pm 2e_i, \pm e_i \pm e_j, i \neq j&e_i-e_{i+1}, 2e_n\\
D_n&\dynkin{D}{n}&\frac{1}{2}\W{n}&\pm e_i \pm e_j, i \neq j&e_i-e_{i+1}, e_n\\
\begin{bunch}e_i-e_{i+1},&e_n-e_{n-1}&e_{n-1}+e_n\end{bunch}\\
E_8&\dynkin{E}{8}&\frac{1}{2}\W{8}&
\begin{bunch}\pm 2e_i \pm 2e_j, &i \neq j, \\
&\sum_i (-1)^{m_i} e_i, &\sum m_i \text{ even}\end{bunch}&
\begin{bunch}2e_1-2e_2, \\
2e_2-2e_3, \\
2e_3-2e_4, \\
2e_4-2e_5, \\
2e_5-2e_6, \\
2e_6+2e_7, \\
-\sum e_j, \\
2e_6-2e_7\end{bunch}
\end{longtable}

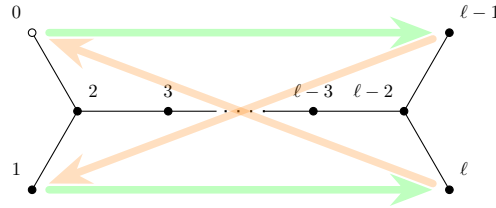
```

```

E_7&\dynkin{E}{7}&\frac{1}{2}\W[e_1-e_2]{8}&\quo&\quo\\
E_6&\dynkin{E}{6}&\frac{1}{3}\W[e_1-e_2,e_2-e_3]{8}&\quo&\quo\\
F_4&\dynkin{F}{4}&\W{4}&
\begin{bunch}\pm 2e_i,\,\,\,\pm 2e_i \pm 2e_j, \quad i \ne j,\,\,\,\pm e_1 \pm e_2 \pm e_3 \pm e_4
\end{bunch}&
\begin{bunch}2e_2-2e_3,\,\,2e_3-2e_4,\,\,2e_4,\,\,e_1-e_2-e_3-e_4\end{bunch}\\
G_2&\dynkin{G}{2}&\W[\sum e_j]{3}&
\begin{bunch}
\pm(1,-1,0),\,\,\,\pm(-1,0,1),\,\,\,\pm(0,-1,1),\,\,\,\pm(2,-1,-1),\,\,\,\pm(1,-2,1),\,\,\,\pm(-1,-1,2)
\end{bunch}&
\begin{bunch}(-1,0,1),\,\,\,(2,-1,-1)\end{bunch}
\end{longtable}

```

## 28. AN EXAMPLE OF MIKHAIL BOROVoi



```

\tikzset{big arrow/.style={
  -Stealth,line cap=round,line width=1mm,
  shorten <=1mm,shorten >=1mm}}
\newcommand\catholic[2]{\draw[big arrow,green!25!white]
(root #1) to (root #2);}
\newcommand\protestant[2]{
\begin{scope}[transparency group, opacity=.25]
\draw[big arrow,orange] (root #1) to (root #2);
\end{scope}}
\begin{dynkinDiagram}[edge length=1.2cm,
indefinite edge/.style={thick,loosely dotted},
labels*={0,1,2,3,\ell-3,\ell-2,\ell-1,\ell}]{D}[1]{
\catholic{0}{6}\catholic{1}{7}
\protestant{7}{0}\protestant{6}{1}
\end{dynkinDiagram}

```

## 29. SYNTAX

The syntax is `\dynkin[<options>]{<letter>}[<twisted rank>]{<rank>}` where `<letter>` is A, B, C, D, E, F or G, the family of root system for the Dynkin diagram, `<twisted rank>` is 0, 1, 2, 3 (default is 0) representing:

- 0 finite root system
- 1 affine extended root system, i.e. of type <sup>(1)</sup>
- 2 affine twisted root system of type <sup>(2)</sup>
- 3 affine twisted root system of type <sup>(3)</sup>

and `<rank>` is

- (1) an integer representing the rank or
- (2) blank to represent an indefinite rank or
- (3) the name of a Satake diagram as in section 4.

The environment syntax is `\begin{dynkinDiagram}` followed by the same parameters as `\dynkin`, then various Dynkin diagram and TikZ commands, and then `\end{dynkinDiagram}`.

## 30. OPTIONS

`ceref` = `<true or false>`,  
 default : `false`  
 whether to draw roots in a “ceref” style.

`edge length` = `<number>cm`,  
 default : `.35cm`  
 distance between nodes in the Dynkin diagram

`edge/.style` = `TikZ style data`,  
 default : `solid,draw=black,fill=white,thin`  
 style of edges in the Dynkin diagram

`Kac` = `<true or false>`,  
 default : `false`  
 whether to draw in the style of [15]

`name` = `<string>`,  
 default : `anonymous`  
 A name for the Dynkin diagram, with `anonymous` treated as a blank; see section 25.

`parabolic` = `<integer>`,  
 default : `0`  
 A parabolic subgroup with specified integer, where the integer is computed as  $n = \sum 2^{i-1}a_i$ ,  $a_i = 0$  or  $1$ , to say that root  $i$  is crossed, i.e. a noncompact root.

`root radius` = `<number>cm`,  
 default : `.05cm`  
 size of the dots and of the crosses in the Dynkin diagram

`text style` = `<TikZ style data>`,  
 default : `scale=.7`  
 Style for any labels on the roots.

`mark` = `<o,O,t,x,X,*>`,

continued ...

Table 23: ...continued

**default** : \*  
         default root mark  
**affine mark** = o,O,t,x,X,\*,  
**default** : \*  
         default root mark for root zero in an affine Dynkin diagram  
**label** = true or false,  
**default** : false  
         whether to label the roots according to the current labelling scheme.  
**label macro** =  $\langle$ 1-parameter  $\text{\TeX}$  macro $\rangle$ ,  
**default** : #1  
         the current labelling scheme for roots.  
**label macro\*** =  $\langle$ 1-parameter  $\text{\TeX}$  macro $\rangle$ ,  
**default** : #1  
         the current labelling scheme for alternate roots.  
**label height** =  $\langle$ 1-parameter  $\text{\TeX}$  macro $\rangle$ ,  
**default** : b  
         the current maximal height of text labels for the roots, set by  
         giving mathematics text of that height.  
**label depth** =  $\langle$ 1-parameter  $\text{\TeX}$  macro $\rangle$ ,  
**default** : g  
         the current maximal depth of text labels for the roots, set by  
         giving mathematics text of that depth.  
**make indefinite edge** =  $\langle$ edge pair  $i$ - $j$  or list of such $\rangle$ ,  
**default** : {}  
         edge pair or list of edge pairs to treat as having indefinitely many  
         roots on them.  
**indefinite edge ratio** =  $\langle$ float $\rangle$ ,  
**default** : 1.6  
         ratio of indefinite edge lengths to other edge lengths.  
**indefinite edge/.style** =  $\langle$ TikZ style data $\rangle$ ,  
**default** : solid,draw=black,fill=white,thin,densely dotted  
         style of the dotted or dashed middle third of each indefinite edge.  
**backwards** =  $\langle$ true or false $\rangle$ ,  
**default** : false  
         whether to reverse right to left.  
**upside down** =  $\langle$ true or false $\rangle$ ,  
**default** : false  
         whether to reverse up to down.  
**arrows** =  $\langle$ true or false $\rangle$ ,  
**default** : true  
         whether to draw the arrows that arise along the edges.  
**reverse arrows** =  $\langle$ true or false $\rangle$ ,  
**default** : true  
         whether to reverse the direction of the arrows that arise along the  
         edges.  
**fold** =  $\langle$ true or false $\rangle$ ,

continued ...

Table 23: ...continued

default : **true**  
           whether, when drawing Dynkin diagrams, to draw them 2-ply.  
 ply =  $\langle 0,1,2,3,4 \rangle$ ,  
 default : 0  
           how many roots get folded together, at most.  
 fold left =  $\langle \text{true or false} \rangle$ ,  
 default : **true**  
           whether to fold the roots on the left side of a Dynkin diagram.  
 fold right =  $\langle \text{true or false} \rangle$ ,  
 default : **true**  
           whether to fold the roots on the right side of a Dynkin diagram.  
 fold radius =  $\langle \text{length} \rangle$ ,  
 default : .3cm  
           the radius of circular arcs used in curved edges of folded Dynkin  
           diagrams.  
 fold style/.style =  $\langle \text{TikZ style data} \rangle$ ,  
 default : **solid,draw=black!40,fill=none,line width=radius**  
           when drawing folded diagrams, style for the fold indicators.  
 \*/.style =  $\langle \text{TikZ style data} \rangle$ ,  
 default : **solid,draw=black,fill=black**  
           style for roots like •  
 o/.style =  $\langle \text{TikZ style data} \rangle$ ,  
 default : **solid,draw=black,fill=black**  
           style for roots like ◦  
 O/.style =  $\langle \text{TikZ style data} \rangle$ ,  
 default : **solid,draw=black,fill=black**  
           style for roots like ⊙  
 t/.style =  $\langle \text{TikZ style data} \rangle$ ,  
 default : **solid,draw=black,fill=black**  
           style for roots like ⊗  
 x/.style =  $\langle \text{TikZ style data} \rangle$ ,  
 default : **solid,draw=black,line cap=round**  
           style for roots like ×  
 X/.style =  $\langle \text{TikZ style data} \rangle$ ,  
 default : **solid,draw=black,thick,line cap=round**  
           style for roots like ✕  
 fold left style/.style =  $\langle \text{TikZ style data} \rangle$ ,  
 default :  
           style to override the fold style when folding roots together on the  
           left half of a Dynkin diagram  
 fold right style/.style =  $\langle \text{TikZ style data} \rangle$ ,  
 default :  
           style to override the fold style when folding roots together on the  
           right half of a Dynkin diagram  
 double edges =  $\langle \rangle$ ,  
 default : **not set**

continued ...

Table 23: ...continued

	set to override the <b>fold</b> style when folding roots together in a Dynkin diagram, so that the foldings are indicated with double edges (like those of an $F_4$ Dynkin diagram without arrows).
<b>double fold</b>	= $\langle \rangle$ ,
default :	<b>not set</b>
	set to override the <b>fold</b> style when folding roots together in a Dynkin diagram, so that the foldings are indicated with double edges (like those of an $F_4$ Dynkin diagram without arrows), but filled in solidly.
<b>double left</b>	= $\langle \rangle$ ,
default :	<b>not set</b>
	set to override the <b>fold</b> style when folding roots together at the left side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an $F_4$ Dynkin diagram without arrows).
<b>double fold left</b>	= $\langle \rangle$ ,
default :	<b>not set</b>
	set to override the <b>fold</b> style when folding roots together at the left side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an $F_4$ Dynkin diagram without arrows), but filled in solidly.
<b>double right</b>	= $\langle \rangle$ ,
default :	<b>not set</b>
	set to override the <b>fold</b> style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an $F_4$ Dynkin diagram without arrows).
<b>double fold right</b>	= $\langle \rangle$ ,
default :	<b>not set</b>
	set to override the <b>fold</b> style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an $F_4$ Dynkin diagram without arrows), but filled in solidly.
<b>arrow color</b>	= $\langle \rangle$ ,
default :	<b>black</b>
	set to override the default color for the arrows in nonsimply laced Dynkin diagrams.
<b>Coxeter</b>	= $\langle \text{true or false} \rangle$ ,
default :	<b>false</b>
	whether to draw a Coxeter diagram, rather than a Dynkin diagram.
<b>ordering</b>	= $\langle \text{Adams, Bourbaki, Carter, Dynkin, Kac} \rangle$ ,
default :	<b>Bourbaki</b>
	which ordering of the roots to use in exceptional root systems as in section 24.

All other options are passed to TikZ.

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