

The `fibnum` package

Heiko Oberdiek*

2016/05/16 v1.1

Abstract

The package `fibnum` provides expandable fibonacci numbers for both \LaTeX and plain \TeX .

Contents

| | | |
|----------|--------------------------------------------|-----------|
| 1 | Documentation | 1 |
| 2 | Implementation | 3 |
| 2.1 | Identification | 3 |
| 2.2 | Package resources | 5 |
| 2.3 | Setup precalculated values | 5 |
| 2.4 | Macros for precalculating values | 6 |
| 2.5 | Expandable calculations | 7 |
| 3 | Installation | 8 |
| 3.1 | Download | 8 |
| 3.2 | Bundle installation | 9 |
| 3.3 | Package installation | 9 |
| 3.4 | Refresh file name databases | 9 |
| 3.5 | Some details for the interested | 9 |
| 4 | References | 10 |
| 5 | History | 10 |
| | [2012/04/08 v1.0] | 10 |
| | [2016/05/16 v1.1] | 10 |
| 6 | Index | 10 |

1 Documentation

In the mailing list `texhax` Jan Abraham asked the question, how to get Fibonacci numbers in \TeX [1]:

Write a Macro in \TeX that compute the function `\fib{m}`. All fibonacci numbers from 1 to m ($m < 40$).

*Please report any issues at <https://github.com/ho-tex/oberdiek/issues>

This packages provides an expandable implementation for the calculation of these numbers for a much larger set of indexes. For practical reasons the index is restricted to the same limitations that apply for T_EX integer numbers. The range of the Fibonacci numbers, however, are not limited by the algorithm. They are only restricted to memory limitations, if they are hit.

The package is loaded as L^AT_EX package in L^AT_EX:

```
\usepackage{fibnum}
```

and as file in plain T_EX:

```
\input fibnum.sty
```

The package does not know any options and it provides the macros `\fibnum` and `\fibnumPreCalc`.

`\fibnum {⟨index⟩}`

Macro `\fibnum` expects a T_EX number as $\langle index \rangle$ in the official T_EX number range from $-(2^{31} - 1)$ up to $2^{31} - 1$. In exact two expansion steps the macro expands to the Fibnoacci number $F_{\langle index \rangle}$. In case of a negative $\langle index \rangle$, the “negafibonacci” number [2] is used. Formally the Fibonacci number F_n with integer index n , $n \in \mathbb{Z}$ and $n \in [-2\,147\,483\,647, 2\,147\,483\,647]$ that is returned by macro `\fibnum` with numerical argument n is defined the following way:

$$F_n = \begin{cases} 0 & \text{for } n = 0 \\ 1 & \text{for } n = 1 \\ F_{n-1} + F_{n-2} & \text{for } n > 1 \\ (-1)^{n+1} F_n & \text{for } n < 0 \end{cases} \quad (1)$$

Examples:

```

\fibnum{-6} → -8
\fibnum{-5} → 5
\fibnum{-4} → -3
\fibnum{-3} → 2
\fibnum{-2} → -1
\fibnum{-1} → 1
\fibnum{0} → 0
\fibnum{1} → 1
\fibnum{2} → 1
\fibnum{3} → 2
\fibnum{4} → 3
\fibnum{5} → 5
\fibnum{6} → 8
⋮
\fibnum{10} → 55
⋮
\fibnum{46} → 1836311903
⋮
\fibnum{100} → 354224848179261915075
⋮
\fibnum{200} → 280571172992510140037611932413038677189525
⋮
\fibnum{1000} → 434665576869374564356885276750406258025646
605173717804024817290895365554179490518904
038798400792551692959225930803226347752096
896232398733224711616429964409065331879382
98969649928516003704476137795166849228875

```

`\fibnumPreCalc {⟨index⟩}`

The package already provides precalculated Fibonacci numbers up to index 46. That means that calculations are not necessary for Fibonacci numbers that fit into the range of $\text{T}_{\text{E}}\text{X}$ numbers. Because macro `\fibnum` is expandable, it cannot store calculated Fibonacci numbers for later use. Macro definitions are forbidden in expandable contexts. If larger Fibonacci numbers are used more than once, then the compilation time can be shortened by calculating and storing the Fibonacci numbers beforehand. The argument $\langle index \rangle$ is a $\text{T}_{\text{E}}\text{X}$ number and macro `\fibnumPreCalc` ensures that the Fibonacci numbers F_0 up to $F_{|\langle index \rangle|}$ that are not already known are calculated and stored in internal macros. Internally only non-negative Fibonacci numbers are stored. If $\langle index \rangle$ is negative, then the needed positive Fibonacci numbers are calculated and stored. Example:

```

\fibnumPreCalc{50}
% calculates and stores the values for indexes 47..50.
% (Values for 0..46 are already stored by the package.)
\fibnum{49} % uses the stored value
\fibnum{51} % only calculates  $F_{51}$  from stored values  $F_{49}$  and  $F_{50}$ 
\fibnumPreCalc{100}
% calculates and stores the values for indexes 51..100
\fibnum{100} % uses the stored value for  $F_{100}$ 
\fibnum{-100} % uses the stored value for  $F_{100}$ 

```

$F_{-100} = -F_{100}$ according to equation (1).

2 Implementation

2.1 Identification

```

1 (*package)
Reload check, especially if the package is not used with LATEX.
2 \begingroup\catcode61\catcode48\catcode32=10\relax%
3 \catcode13=5 % ^M
4 \endlinechar=13 %
5 \catcode35=6 % #
6 \catcode39=12 % '
7 \catcode44=12 % ,
8 \catcode45=12 % -
9 \catcode46=12 % .
10 \catcode58=12 % :
11 \catcode64=11 % @
12 \catcode123=1 % {
13 \catcode125=2 % }
14 \expandafter\let\expandafter\x\csname ver@fibnum.sty\endcsname
15 \ifx\x\relax % plain-TeX, first loading
16 \else
17 \def\empty{}%
18 \ifx\x\empty % LaTeX, first loading,
19 % variable is initialized, but \ProvidesPackage not yet seen
20 \else
21 \expandafter\ifx\csname PackageInfo\endcsname\relax
22 \def\x#1#2{%
23 \immediate\write-1{Package #1 Info: #2.}%
24 }%
25 \else
26 \def\x#1#2{\PackageInfo{#1}{#2, stopped}}%
27 \fi
28 \x{fibnum}{The package is already loaded}%
29 \aftergroup\endinput
30 \fi
31 \fi
32 \endgroup%
Package identification:
33 \begingroup\catcode61\catcode48\catcode32=10\relax%
34 \catcode13=5 % ^M
35 \endlinechar=13 %
36 \catcode35=6 % #
37 \catcode39=12 % '
38 \catcode40=12 % (
39 \catcode41=12 % )
40 \catcode44=12 % ,
41 \catcode45=12 % -
42 \catcode46=12 % .
43 \catcode47=12 % /
44 \catcode58=12 % :
45 \catcode64=11 % @
46 \catcode91=12 % [
47 \catcode93=12 % ]
48 \catcode123=1 % {
49 \catcode125=2 % }

```

```

50 \expandafter\ifx\cscname ProvidesPackage\endcscname\relax
51 \def\x#1#2#3[#4]{\endgroup
52 \immediate\write-1{Package: #3 #4}%
53 \xdef#1{#4}%
54 }%
55 \else
56 \def\x#1#2[#3]{\endgroup
57 #2[{#3}]%
58 \ifx#1\@undefined
59 \xdef#1{#3}%
60 \fi
61 \ifx#1\relax
62 \xdef#1{#3}%
63 \fi
64 }%
65 \fi
66 \expandafter\x\cscname ver@fibnum.sty\endcscname
67 \ProvidesPackage{fibnum}%
68 [2016/05/16 v1.1 Fibonacci numbers (HO)]%
69 \begingroup\catcode61\catcode48\catcode32=10\relax%
70 \catcode13=5 % ^~M
71 \endlinechar=13 %
72 \catcode123=1 % {
73 \catcode125=2 % }
74 \catcode64=11 % @
75 \def\x{\endgroup
76 \expandafter\edef\cscname FibNum@AtEnd\endcscname{%
77 \endlinechar=\the\endlinechar\relax
78 \catcode13=\the\catcode13\relax
79 \catcode32=\the\catcode32\relax
80 \catcode35=\the\catcode35\relax
81 \catcode61=\the\catcode61\relax
82 \catcode64=\the\catcode64\relax
83 \catcode123=\the\catcode123\relax
84 \catcode125=\the\catcode125\relax
85 }%
86 }%
87 \x\catcode61\catcode48\catcode32=10\relax%
88 \catcode13=5 % ^~M
89 \endlinechar=13 %
90 \catcode35=6 % #
91 \catcode64=11 % @
92 \catcode123=1 % {
93 \catcode125=2 % }
94 \def\TMP@EnsureCode#1#2{%
95 \edef\FibNum@AtEnd{%
96 \FibNum@AtEnd
97 \catcode#1=\the\catcode#1\relax
98 }%
99 \catcode#1=#2\relax
100 }
101 \TMP@EnsureCode{33}{12}% !
102 %\TMP@EnsureCode{36}{3}% $
103 %\TMP@EnsureCode{38}{4}% &
104 \TMP@EnsureCode{40}{12}% (
105 \TMP@EnsureCode{41}{12}% )
106 \TMP@EnsureCode{45}{12}% -
107 \TMP@EnsureCode{46}{12}% .

```

```

108 \TMP@EnsureCode{47}{12}% /
109 \TMP@EnsureCode{58}{12}% :
110 \TMP@EnsureCode{60}{12}% <
111 \TMP@EnsureCode{62}{12}% >
112 \TMP@EnsureCode{91}{12}% [
113 \% \TMP@EnsureCode{96}{12}% '
114 \TMP@EnsureCode{93}{12}% ]
115 \% \TMP@EnsureCode{94}{12}% ^ (superscript) (!)
116 \% \TMP@EnsureCode{124}{12}% |
117 \edef\FibNum@AtEnd{\FibNum@AtEnd\noexpand\endinput}

```

2.2 Package resources

```

118 \begingroup\expandafter\expandafter\expandafter\endgroup
119 \expandafter\ifx\csname RequirePackage\endcsname\relax
120   \def\TMP@RequirePackage#1[#2]{%
121     \begingroup\expandafter\expandafter\expandafter\endgroup
122     \expandafter\ifx\csname ver@#1.sty\endcsname\relax
123       \input #1.sty\relax
124     \fi
125   }%
126   \TMP@RequirePackage{ltxcms}[2011/04/18]%
127   \TMP@RequirePackage{intcalc}[2007/09/27]%
128   \TMP@RequirePackage{bigintcalc}[2007/11/11]%
129 \else
130   \RequirePackage{ltxcms}[2011/04/18]%
131   \RequirePackage{intcalc}[2007/09/27]%
132   \RequirePackage{bigintcalc}[2007/11/11]%
133 \fi

```

2.3 Setup precalculated values

```

134 \def\FibNum@temp#1{%
135   \expandafter\def\csname FibNum@#1\endcsname
136 }
137 \catcode46=9 % dots are ignored
138 \FibNum@temp{0}{0}
139 \FibNum@temp{1}{1}
140 \FibNum@temp{2}{1}
141 \FibNum@temp{3}{2}
142 \FibNum@temp{4}{3}
143 \FibNum@temp{5}{5}
144 \FibNum@temp{6}{8}
145 \FibNum@temp{7}{13}
146 \FibNum@temp{8}{21}
147 \FibNum@temp{9}{34}
148 \FibNum@temp{10}{55}
149 \FibNum@temp{11}{89}
150 \FibNum@temp{12}{144}
151 \FibNum@temp{13}{233}
152 \FibNum@temp{14}{377}
153 \FibNum@temp{15}{610}
154 \FibNum@temp{16}{987}
155 \FibNum@temp{17}{1.597}
156 \FibNum@temp{18}{2.584}
157 \FibNum@temp{19}{4.181}
158 \FibNum@temp{20}{6.765}
159 \FibNum@temp{21}{10.946}
160 \FibNum@temp{22}{17.711}
161 \FibNum@temp{23}{28.657}

```

```

162 \FibNum@temp{24}{46.368}
163 \FibNum@temp{25}{75.025}
164 \FibNum@temp{26}{121.393}
165 \FibNum@temp{27}{196.418}
166 \FibNum@temp{28}{317.811}
167 \FibNum@temp{29}{514.229}
168 \FibNum@temp{30}{832.040}
169 \FibNum@temp{31}{1.346.269}
170 \FibNum@temp{32}{2.178.309}
171 \FibNum@temp{33}{3.524.578}
172 \FibNum@temp{34}{5.702.887}
173 \FibNum@temp{35}{9.227.465}
174 \FibNum@temp{36}{14.930.352}
175 \FibNum@temp{37}{24.157.817}
176 \FibNum@temp{38}{39.088.169}
177 \FibNum@temp{39}{63.245.986}
178 \FibNum@temp{40}{102.334.155}
179 \FibNum@temp{41}{165.580.141}
180 \FibNum@temp{42}{267.914.296}
181 \FibNum@temp{43}{433.494.437}
182 \FibNum@temp{44}{701.408.733}
183 \FibNum@temp{45}{1.134.903.170}
184 \FibNum@temp{46}{1.836.311.903}

\FibNum@max

185 \def\FibNum@max{46}

```

2.4 Macros for precalculating values

```

\fibnumPreCalc

186 \def\fibnumPreCalc#1{%
187   \expandafter\expandafter\expandafter
188   \FibNum@PreCalc\intcalcNum{#1}/%
189 }

\FibNum@PreCalc

190 \def\FibNum@PreCalc#1/{%
191   \ifnum#1<\ltx@zero
192     \expandafter\FibNum@PreCalc\ltx@gobble#1/%
193   \else
194     \ifnum#1>\FibNum@max
195       \begingroup
196         \ltx@LocDimenA=#1sp\relax
197         \countdef\FibNum@i=255\relax
198         \FibNum@i=\FibNum@max\relax
199         \edef\FibNum@temp{%
200           \csname FibNum@the\FibNum@i\endcsname/%
201         }%
202         \advance\FibNum@i by -1\relax
203         \edef\FibNum@temp{%
204           \FibNum@temp
205           \csname FibNum@the\FibNum@i\endcsname
206         }%
207         \advance\FibNum@i\ltx@two
208         \iftrue
209           \expandafter\FibNum@PreCalcAux\FibNum@temp
210         \fi
211       \endgroup

```

```

212   \fi
213   \fi
214 }

\FibNum@PreCalcAux

215 \def\FibNum@PreCalcAux#1/#2\fi{%
216   \fi
217   \edef\FibNum@temp{\BigIntCalcAdd#1!#2!}%
218   \global\expandafter
219   \let\csname FibNum@\the\FibNum@i\endcsname\FibNum@temp
220   \ifnum\FibNum@i=\ltx@LocDimenA
221     \xdef\FibNum@max{\the\FibNum@i}%
222   \else
223     \advance\FibNum@i\ltx@one
224     \expandafter\FibNum@PreCalcAux\FibNum@temp/#1%
225   \fi
226 }

```

2.5 Expandable calculations

```

\fibnum

227 \def\fibnum#1{%
228   \romannumeral
229   \expandafter\expandafter\expandafter\FibNum@Do\intcalcNum{#1}/%
230 }

\FibNum@Do

231 \def\FibNum@Do#1/{%
232   \ifnum#1<\ltx@zero
233     \FibNum@ReturnAfterElseFiFi{%
234       \ifodd#1 %
235         \expandafter\expandafter\expandafter\ltx@zero
236       \else
237         \expandafter\expandafter\expandafter\ltx@zero
238         \expandafter\expandafter\expandafter~%
239       \fi
240       \romannumeral
241       \expandafter\FibNum@Do\ltx@gobble#1/%
242     }%
243   \else
244     \ifnum\FibNum@max<#1 %
245       \ltx@ReturnAfterElseFiFi{%
246         \expandafter
247         \FibNum@ExpCalc\number\expandafter\IntCalcInc\FibNum@max!%
248         \expandafter\expandafter\expandafter/%
249         \csname FibNum@\FibNum@max
250         \expandafter\expandafter\expandafter\endcsname
251         \expandafter\expandafter\expandafter/%
252         \csname FibNum@\expandafter\IntCalcDec\FibNum@max!%
253         \endcsname/%
254         #1%
255       }%
256     \else
257       \expandafter\expandafter\expandafter\ltx@zero
258       \csname FibNum@#1\expandafter\expandafter\expandafter\endcsname
259     \fi
260   \fi
261 }

```



```

\FibNum@ReturnAfterElseFiFi
262 \def\FibNum@ReturnAfterElseFiFi#1\else#2\fi\fi{\fi#1}

\FibNum@ExpCalc
263 \def\FibNum@ExpCalc#1/#2/#3/#4\fi{%
264   \fi
265   \ifnum#1=#4 %
266     \ltx@ReturnAfterElseFi{%
267       \expandafter\expandafter\expandafter\ltx@zero
268       \BigIntCalcAdd#2!#3!%
269     }%
270   \else
271     \expandafter\FibNum@ExpCalc
272     \number\IntCalcInc#1!%
273     \expandafter\expandafter\expandafter/%
274     \BigIntCalcAdd#2!#3!/%
275     #2/#4%
276   \fi
277 }

278 \FibNum@AtEnd%
279 \</package>

```

3 Installation

3.1 Download

Package. This package is available on CTAN¹:

[CTAN:macros/latex/contrib/oberdiek/fibnum.dtx](#) The source file.

[CTAN:macros/latex/contrib/oberdiek/fibnum.pdf](#) Documentation.

Bundle. All the packages of the bundle ‘oberdiek’ are also available in a TDS compliant ZIP archive. There the packages are already unpacked and the documentation files are generated. The files and directories obey the TDS standard.

[CTAN:install/macros/latex/contrib/oberdiek.tds.zip](#)

TDS refers to the standard “A Directory Structure for T_EX Files” ([CTAN:pkg/tds](#)). Directories with `texmf` in their name are usually organized this way.

3.2 Bundle installation

Unpacking. Unpack the `oberdiek.tds.zip` in the TDS tree (also known as `texmf` tree) of your choice. Example (linux):

```
unzip oberdiek.tds.zip -d ~/texmf
```

3.3 Package installation

Unpacking. The `.dtx` file is a self-extracting docstrip archive. The files are extracted by running the `.dtx` through plain T_EX:

```
tex fibnum.dtx
```

¹[CTAN:pkg/fibnum](#)

TDS. Now the different files must be moved into the different directories in your installation TDS tree (also known as `texmf` tree):

```
fibnum.sty → tex/generic/oberdiek/fibnum.sty
fibnum.pdf → doc/latex/oberdiek/fibnum.pdf
fibnum.dtx → source/latex/oberdiek/fibnum.dtx
```

If you have a `docstrip.cfg` that configures and enables `docstrip`'s TDS installing feature, then some files can already be in the right place, see the documentation of `docstrip`.

3.4 Refresh file name databases

If your $\text{T}_{\text{E}}\text{X}$ distribution ($\text{T}_{\text{E}}\text{X}$ Live, `mikTEX`, ...) relies on file name databases, you must refresh these. For example, $\text{T}_{\text{E}}\text{X}$ Live users run `texhash` or `mktextlsr`.

3.5 Some details for the interested

Unpacking with $\text{L}^{\text{A}}\text{T}_{\text{E}}\text{X}$. The `.dtx` chooses its action depending on the format:

plain $\text{T}_{\text{E}}\text{X}$: Run `docstrip` and extract the files.

$\text{L}^{\text{A}}\text{T}_{\text{E}}\text{X}$: Generate the documentation.

If you insist on using $\text{L}^{\text{A}}\text{T}_{\text{E}}\text{X}$ for `docstrip` (really, `docstrip` does not need $\text{L}^{\text{A}}\text{T}_{\text{E}}\text{X}$), then inform the autodetect routine about your intention:

```
latex \let\install=y\input{fibnum.dtx}
```

Do not forget to quote the argument according to the demands of your shell.

Generating the documentation. You can use both the `.dtx` or the `.drv` to generate the documentation. The process can be configured by the configuration file `ltxdoc.cfg`. For instance, put this line into this file, if you want to have A4 as paper format:

```
\PassOptionsToClass{a4paper}{article}
```

An example follows how to generate the documentation with `pdfLATEX`:

```
pdflatex fibnum.dtx
bibtex fibnum.aux
makeindex -s gind.ist fibnum.idx
pdflatex fibnum.dtx
makeindex -s gind.ist fibnum.idx
pdflatex fibnum.dtx
```

4 References

- [1] Jan Abraham. *[texhax] Beginner in TEX MACRO to compute functions*. 2012-04-07. URL: <https://tug.org/pipermail/texhax/2012-April/019146.html> (visited on 2012-04-08).
- [2] Wikipedia contributors. *Fibonacci numbers*. Version 486266088. Wikipedia, The Free Encyclopedia. 2012-04-08. URL: https://en.wikipedia.org/w/index.php?title=Fibonacci_number&oldid=486266088 (visited on 2012-04-08).

5 History

[2012/04/08 v1.0]

- First version.

[2016/05/16 v1.1]

- Documentation updates.

6 Index

Numbers written in *italic* refer to the page where the corresponding entry is described; numbers underlined refer to the code line of the definition; plain numbers refer to the code lines where the entry is used.

| Symbols | | 143, 144, 145, 146, 147, 148, |
|-----------------------------|-------------------------------------|--------------------------------------------|
| \@undefined | 58 | 149, 150, 151, 152, 153, 154, |
| | | 155, 156, 157, 158, 159, 160, |
| A | | 161, 162, 163, 164, 165, 166, |
| \advance | 202, 207, 223 | 167, 168, 169, 170, 171, 172, |
| \aftergroup | 29 | 173, 174, 175, 176, 177, 178, |
| | | 179, 180, 181, 182, 183, 184, |
| B | | 199, 203, 204, 209, 217, 219, 224 |
| \BigIntCalcAdd | 217, 268, 274 | \fibnumPreCalc 3, <u>186</u> |
| C | | I |
| \catcode | 2, 3, | \ifnum 191, 194, 220, 232, 244, 265 |
| | 5, 6, 7, 8, 9, 10, 11, 12, 13, 33, | \ifodd 234 |
| | 34, 36, 37, 38, 39, 40, 41, 42, 43, | \iftrue 208 |
| | 44, 45, 46, 47, 48, 49, 69, 70, 72, | \ifx 15, 18, 21, 50, 58, 61, 119, 122 |
| | 73, 74, 78, 79, 80, 81, 82, 83, 84, | \immediate 23, 52 |
| | 87, 88, 90, 91, 92, 93, 97, 99, 137 | \input 123 |
| \countdef | 197 | \IntCalcDec 252 |
| \csname | 14, 21, 50, 66, 76, 119, 122, | \IntCalcInc 247, 272 |
| | 135, 200, 205, 219, 249, 252, 258 | \intcalcNum 188, 229 |
| E | | L |
| \empty | 17, 18 | \ltx@gobble 192, 241 |
| \endcsname | 14, 21, 50, 66, 76, 119, 122, | \ltx@LocDimenA 196, 220 |
| | 135, 200, 205, 219, 250, 253, 258 | \ltx@one 223 |
| \endinput | 29, 117 | \ltx@ReturnAfterElseFi 245, 266 |
| \endlinechar | 4, 35, 71, 77, 89 | \ltx@two 207 |
| F | | \ltx@zero . 191, 232, 235, 237, 257, 267 |
| \fibnum | 2, <u>227</u> | N |
| \FibNum@AtEnd | 95, 96, 117, 278 | \number 247, 272 |
| \FibNum@Do | 229, <u>231</u> | P |
| \FibNum@ExpCalc | 247, <u>263</u> | \PackageInfo 26 |
| \FibNum@i | 197, 198, 200, | \ProvidesPackage 19, 67 |
| | 202, 205, 207, 219, 220, 221, 223 | R |
| \FibNum@max | <u>185</u> , | \RequirePackage 130, 131, 132 |
| | 194, 198, 221, 244, 247, 249, 252 | \romannumeral 228, 240 |
| \FibNum@PreCalc | 188, <u>190</u> | T |
| \FibNum@PreCalcAux | 209, <u>215</u> | \the 77, 78, 79, 80, |
| \FibNum@ReturnAfterElseFiFi | 233, <u>262</u> | 81, 82, 83, 84, 97, 200, 205, 219, 221 |
| \FibNum@temp | | |
| | 134, 138, 139, 140, 141, 142, | |

| | | | | |
|---------------------|---|-----------------------------------|--------------|--------------------------------------------|
| \TMP@EnsureCode | . | 94, 101, 102, 103, | | W |
| | | 104, 105, 106, 107, 108, 109, | \write | 23, 52 |
| | | 110, 111, 112, 113, 114, 115, 116 | | |
| | | | | X |
| \TMP@RequirePackage | | 120, 126, 127, 128 | \x | 14, 15, 18, 22, 26, 28, 51, 56, 66, 75, 87 |