

THE DYNKIN DIAGRAMS PACKAGE
VERSION 3.14159265358

BEN MCKAY

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1. QUICK INTRODUCTION

Load the Dynkin diagram package (see options below)

```
\documentclass{amsart}
\usepackage{dynkin-diagrams}
\begin{document}
The Dynkin diagram of  $(B_3)$  is \dynkin B3.
\end{document}
```

Invoke it

The Dynkin diagram of (B_3) is \dynkin B3.

The Dynkin diagram of B_3 is $\bullet \rightarrow \bullet$.

Inside a TikZ statement

The Dynkin diagram of (B_3) is
 $\tikz \dynkin B3;$

The Dynkin diagram of B_3 is $\bullet \rightarrow \bullet$

Inside a Dynkin diagram environment

The Dynkin diagram of (B_3) is
 $\begin{dynkinDiagram} B3 \\ \draw[very thick,red] (root 1) to [out=-45, in=-135] (root 3); \\ \end{dynkinDiagram}$

The Dynkin diagram of B_3 is $\bullet \rightarrow \bullet$

Inside a TikZ environment

Baseline controls vertical alignment:
the Dynkin diagram of (B_3) is
 $\begin{tikzpicture}[baseline=(origin.base)] \\ \dynkin B3 \\ \draw[very thick,red] (root 1) to [out=-45, in=-135] (root 3); \\ \end{tikzpicture}$

Baseline controls vertical alignment: the Dynkin diagram of B_3 is $\bullet \rightarrow \bullet$

In a TikZ picture, you might need to kill the default vertical shift (needed to allow inline Dynkin diagrams):

Inside TikZ pictures

```
\begin{tikzpicture}
\draw (0,0) -- (.5,1) -- (1,0);
\dynkin[vertical shift=0,edge length=1cm]G2
\end{tikzpicture}
```



Indefinite rank Dynkin diagrams

```
\dynkin B{}
```



Table 1: The Dynkin diagrams of the reduced simple root systems
[3] pp. 265–290, plates I–IX

A_n		<code>\dynkin A{}</code>
C_n		<code>\dynkin C{}</code>
D_n		<code>\dynkin D{}</code>
E_6		<code>\dynkin E6</code>
E_7		<code>\dynkin E7</code>
E_8		<code>\dynkin E8</code>
F_4		<code>\dynkin F4</code>
G_2		<code>\dynkin G2</code>

2. SET OPTIONS GLOBALLY

Most options set globally ...

```
\pgfkeys{/Dynkin diagram,edge length=.5cm,fold radius=.5cm,
indefinite edge/.style={
draw=black,fill=white,thin,densely dashed}}
```

You can also pass options to the package in `\usepackage`. *Danger*: spaces in option names are replaced with hyphens: `edge length=1cm` is `edge-length=1cm` as a global option; moreover you should drop the extension `/.style` on any option with spaces in its name (but not otherwise). For example,

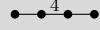
...or pass global options to the package

```
\usepackage[
  ordering=Kac,
  edge/.style=blue,
  indefinite-edge={draw=green,fill=white,densely dashed},
  indefinite-edge-ratio=5,
  mark=o,
  root-radius=.06cm]
{dynkin-diagrams}
```

3. COXETER DIAGRAMS

Coxeter diagram option

```
\dynkin[Coxeter]{F}{4}
```



gonality option for G_2 and I_n Coxeter diagrams

```
\(G_2=\dynkin[Coxeter,gonality=n]G2\), \
\ (I_n=\dynkin[Coxeter,gonality=n]I{\})
```


$G_2 = \overset{n}{\bullet}\bullet$, $I_n = \bullet\overset{n}{\bullet}$

Table 2: The Coxeter diagrams of the simple reflection groups

A_n		<code>\dynkin[Coxeter]A{}</code>
B_n		<code>\dynkin[Coxeter]B{}</code>
C_n		<code>\dynkin[Coxeter]C{}</code>
E_6		<code>\dynkin[Coxeter]E6</code>
E_7		<code>\dynkin[Coxeter]E7</code>
E_8		<code>\dynkin[Coxeter]E8</code>
F_4		<code>\dynkin[Coxeter]F4</code>
G_2		<code>\dynkin[Coxeter,gonality=n]G2</code>
H_3		<code>\dynkin[Coxeter]H3</code>
H_4		<code>\dynkin[Coxeter]H4</code>

continued ...


Table 2: ...continued

I_n		<code>\dynkin[Coxeter,gonality=n]I{}</code>
-------	---	---

4. SATAKE DIAGRAMS

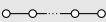

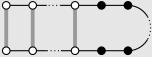
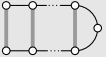

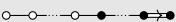
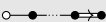

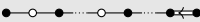
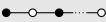




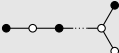
Satake diagrams use the standard name instead of a rank

`\(A_{IIIb}=\dynkin A{IIIb}\)`

$$A_{IIIb} = \text{Diagram}$$


We use a solid gray bar to denote the folding of a Dynkin diagram, rather than the usual double arrow, since the diagrams turn out simpler and easier to read.

Table 3: The Satake diagrams of the real simple Lie algebras [13] p. 532–534

A_I		<code>\dynkin AI</code>
A_{II}		<code>\dynkin A{II}</code>
A_{IIIa}		<code>\dynkin A{IIIa}</code>
A_{IIIb}		<code>\dynkin A{IIIb}</code>
A_{IV}		<code>\dynkin A{IV}</code>
B_I		<code>\dynkin BI</code>
B_{II}		<code>\dynkin B{II}</code>
C_I		<code>\dynkin CI</code>
C_{IIa}		<code>\dynkin C{IIa}</code>
C_{IIb}		<code>\dynkin C{IIb}</code>
D_{Ia}		<code>\dynkin D{Ia}</code>
D_{Ib}		<code>\dynkin D{Ib}</code>
D_{Ic}		<code>\dynkin D{Ic}</code>
D_{II}		<code>\dynkin D{II}</code>
D_{IIIa}		<code>\dynkin D{IIIa}</code>

continued ...

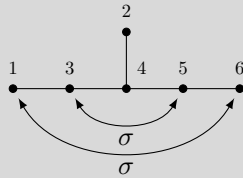
Table 3: ...continued

D_{IIIb}		<code>\dynkin D{IIIb}</code>
E_I		<code>\dynkin E{I}</code>
E_{II}		<code>\dynkin E{II}</code>
E_{III}		<code>\dynkin E{III}</code>
E_{IV}		<code>\dynkin E{IV}</code>
E_V		<code>\dynkin E{V}</code>
E_{VI}		<code>\dynkin E{VI}</code>
E_{VII}		<code>\dynkin E{VII}</code>
E_{VIII}		<code>\dynkin E{VIII}</code>
E_{IX}		<code>\dynkin E{IX}</code>
F_I		<code>\dynkin F{I}</code>
F_{II}		<code>\dynkin F{II}</code>
G_I		<code>\dynkin G{I}</code>

5. HOW TO FOLD

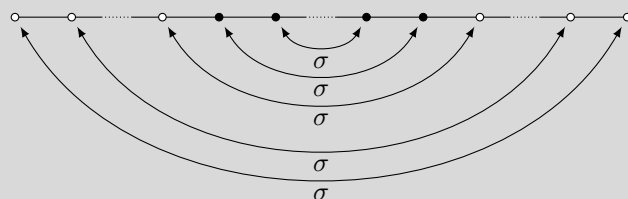
If you don't like the solid gray "folding bar", most people use arrows. Here is E_{II}

```
\newcommand{\invol}[2]{\draw[latex-latex] (root #1) to
[out=-60,in=-120] node[midway,below]{ $\sigma$ } (root #2);}
\begin{dynkinDiagram}[edge length=.75cm,labels*={1,...,6}]E6
\invol 16\invol 35
\end{dynkinDiagram}
```



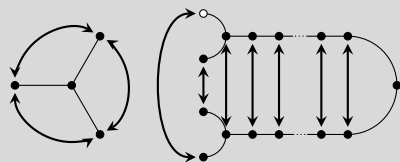
The double arrows for A_{IIIa} are big

```
\newcommand{\invol}[2]{\draw[latex-latex] (root #1) to
[out=-60,in=-120] node[midway,below]{ $\sigma$ } (root #2);}
\begin{dynkinDiagram}[edge length=.75cm]{A}{oo.o**.oo}
\invol 1{10}\invol 29\invol 38\invol 47\invol 56
\end{dynkinDiagram}
```



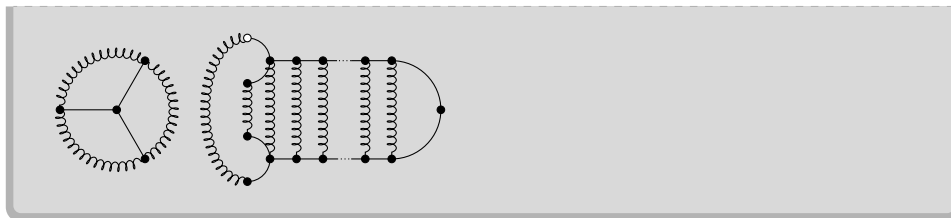
If you don't like the solid gray "folding bar", most people use arrows ...

```
\tikzset{/Dynkin diagram/fold style/.style={stealth-stealth,thick,
shorten <=1mm,shorten >=1mm,}}
\dynkin[ply=3,edge length=.75cm]D4
\begin{dynkinDiagram}[ply=4]D[1]%
{****.*****.*****}
\dynkinFold 1{13}
\dynkinFold[bend right=90] 0{14}
\end{dynkinDiagram}
```



... but you could try springs pulling roots together

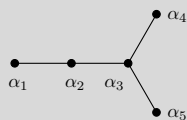
```
\tikzset{/Dynkin diagram/fold style/.style=
{decorate,decoration={name=coil,aspect=0.5,
segment length=1mm,amplitude=.6mm}}}
\dynkin[ply=3,edge length=.75cm]D4
\begin{dynkinDiagram}[ply=4]D[1]%
{****.*****.*****}
\dynkinFold 1{13}
\dynkinFold[bend right=90] 0{14}
\end{dynkinDiagram}
```



6. LABELS FOR THE ROOTS

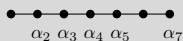
Make a macro to assign labels to roots

```
\dynkin[label,label macro/.code={\alpha_{\drlap{#1}}},edge
length=.75cm]D5
```



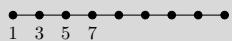
Labelling several roots

```
\dynkin[labels={,2,...,5,,7},label macro/.code={\alpha_{\drlap{#1}}}]A7
```



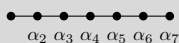
The foreach notation I

```
\dynkin[labels={1,3,...,7},]A9
```



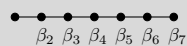
The foreach notation II

```
\dynkin[labels={,\alpha_2,\alpha_....,\alpha_7},]A7
```



The foreach notation III

```
\dynkin[label macro/.code={\beta_{\drlap{#1}}},labels={,2,...,7},]A7
```

Label the roots individually by root number

```
\dynkin[label]B3
```



Label a single root

```
\begin{dynkinDiagram}B3
\dynkinLabelRoot 2{\alpha_{\drlap{2}}}
\end{dynkinDiagram}
```



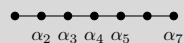
Access root labels via TikZ

```
\begin{dynkinDiagram}B3
\node[below] at (root 2) {\(\alpha_{\drlap{2}}\)};
\end{dynkinDiagram}
```



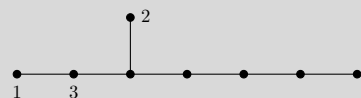
Commands to label several roots

```
\begin{dynkinDiagram}A7
\dynkinLabelRoots{\alpha_2,\alpha_3,\alpha_4,\alpha_5,,\alpha_7}
\end{dynkinDiagram}
```



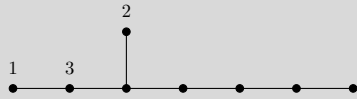
The labels have default locations, mostly below roots

```
\dynkin[edge length=.75cm,labels={1,2,3}]E8
```



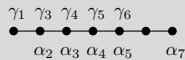
The starred form flips labels to alternate locations, mostly above roots

```
\dynkin[edge length=.75cm,labels*={1,2,3}]E8
```



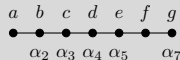
Labelling several roots and alternates

```
\dynkin[%
label macro/.code={\alpha_{\drlap{#1}}},
label macro*/.code={\gamma_{\drlap{#1}}},
labels={,2,...,5,,7},
labels*={1,3,4,5,6}]A7
```



Commands to label several roots

```
\begin{dynkinDiagram}A7
\dynkinLabelRoots{,\alpha_2,\alpha_3,\alpha_4,\alpha_5,,\alpha_7}
\dynkinLabelRoots*{a,b,c,d,e,f,g}
\end{dynkinDiagram}
```

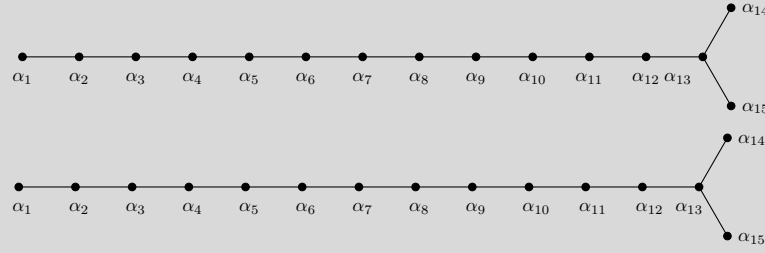


7. LABEL SUBSCRIPTS

Note the slight improvement that `\drlap` makes: the labels are centered on the middle of the letter α , ignoring the space taken up by the subscripts, using the `mathtools` command `\mathrlap`, but only for labels which are *not* placed to the left or right of a root.

Label subscript spacing

```
\dynkin[label,label macro/.code={\alpha_{#1}},
edge length=.75cm]D{15}
\par\noindent{%
\dynkin[label,label macro/.code={\alpha_{\drlap{#1}}},
edge length=.75cm]D{15}
```

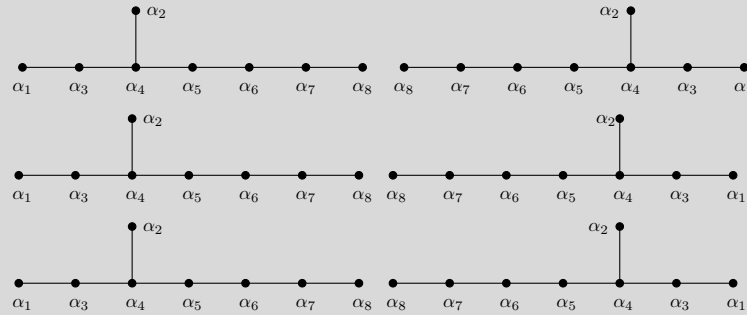


Label subscript spacing

```

\dynkin[label,label macro/.code={\alpha_{#1}},
  edge length=.75cm]E8
\dynkin[label,label macro/.code={\alpha_{#1}},backwards,
  edge length=.75cm]E8
\par\noindent{}%
\dynkin[label,label macro/.code={\alpha_{\mathrlap{#1}}},
  edge length=.75cm]E8
\dynkin[label,label macro/.code={\alpha_{\mathrlap{#1}}},backwards,
  edge length=.75cm]E8
\par\noindent{}%
\dynkin[label,label macro/.code={\alpha_{\drlap{#1}}},
  edge length=.75cm]E8
\dynkin[label,label macro/.code={\alpha_{\drlap{#1}}},backwards,
  edge length=.75cm]E8

```

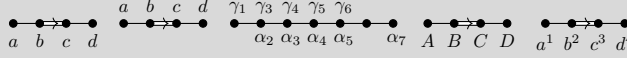


8. HEIGHT AND DEPTH OF LABELS

Labels are set with default maximum height the height of the character b , and default maximum depth the depth of the character g . To change these, set label height and label depth:

Change height and depth of characters

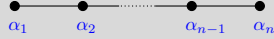
```
\dynkin[labels={a,b,c,d},label height=d,label depth=d]F4
\dynkin[labels*={a,b,c,d},label height=d,label depth=d]F4
\dynkin[%
label macro/.code={\alpha_{\drlap{#1}}},
label macro*/.code={\gamma_{\drlap{#1}}},
label height=$\alpha_1$,
label depth=$\alpha_1$,
labels={,2,...,5,,7},
labels*={1,3,4,5,6}]A7
\dynkin[labels={A,B,C,D},label height=$A$,label depth=$A$]F4
\dynkin[labels={a^1,b^2,c^3,d^4},label height=$X^X$]F4
```



9. TEXT STYLE FOR THE LABELS

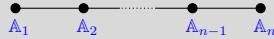
Use a text style: big and blue

```
\begin{dynkinDiagram}[text style={scale=1.2,blue},
edge length=.75cm,
labels={1,2,n-1,n},
label macro/.code={\alpha_{\drlap{#1}}}
]A{}
\end{dynkinDiagram}
```



Use a text style; font selection is in the label macro

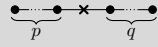
```
\begin{dynkinDiagram}[text style={scale=1.2,blue},
edge length=.75cm,
labels={1,2,n-1,n},
label macro/.code={\mathbb{A}_{\drlap{#1}}}
]A{}
\end{dynkinDiagram}
```



10. BRACING ROOTS

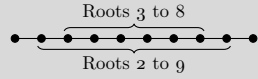
Bracing roots

```
\begin{dynkinDiagram}A{*.x*.*}
\dynkinBrace[p]12
\dynkinBrace[q]45
\end{dynkinDiagram}
```



Bracing roots, and a starred form

```
\begin{dynkinDiagram}A{10}
\dynkinBrace[\text{Roots 2 to 9}]29
\dynkinBrace*[\text{Roots 3 to 8}]38
\end{dynkinDiagram}
```



Bracing roots

```
\newcommand\circleRoot[1]{\draw (root #1) circle (3pt);}
\begin{dynkinDiagram}A{**,***,***,***,***,**}
\circleRoot 4\circleRoot 7\circleRoot 10\circleRoot 13
\dynkinBrace[y-1]13
\dynkinBrace[z-1]56
\dynkinBrace[t-1]{11}{12}
\dynkinBrace[x-1]{14}{16}
\end{dynkinDiagram}
```

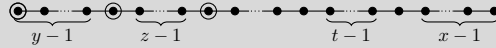
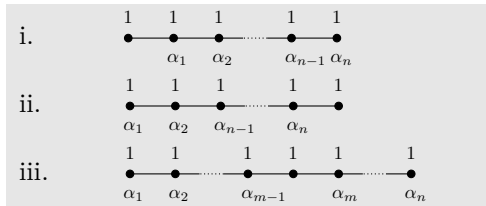
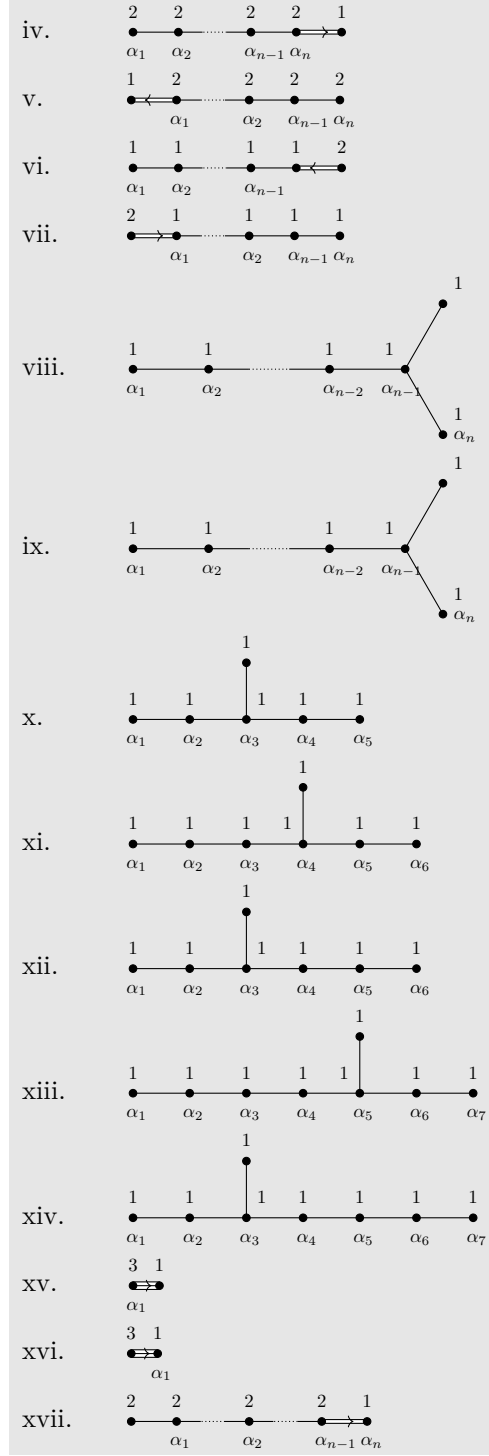


Table 4: Dynkin diagrams from Euler products [17]



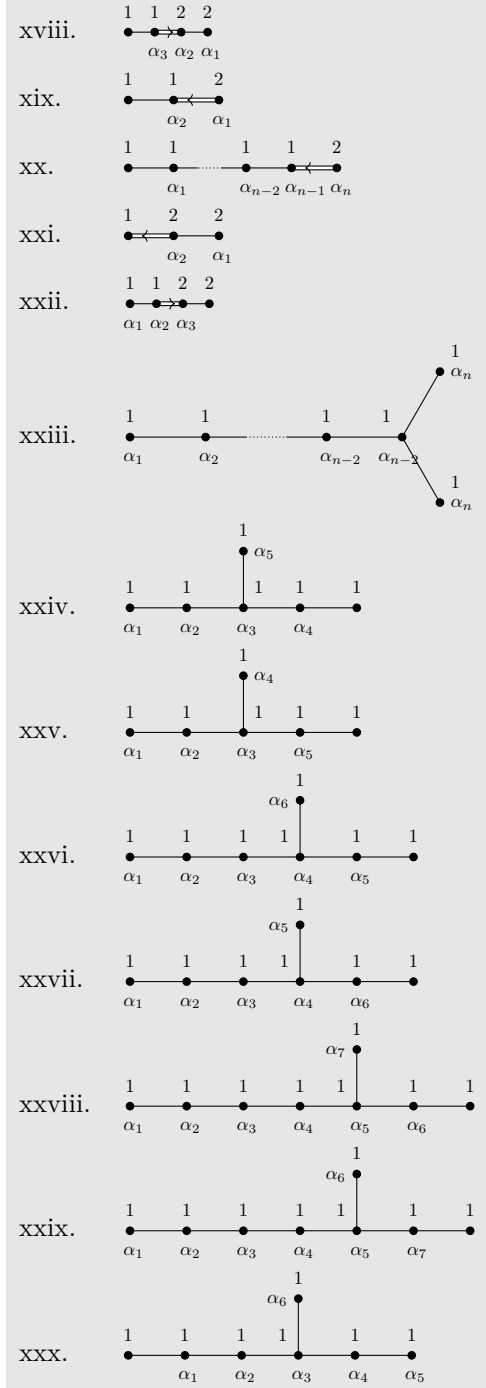
continued ...

Table 4: ...continued



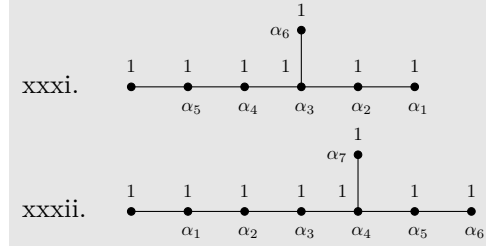
continued ...

Table 4: ...continued



continued ...

Table 4: ...continued



```

\tikzset{/Dynkin diagram,ordering=Dynkin,label macro/.code={\alpha_{\drlap{#1}}}}
\newcounter{EPNo}
\setcounter{EPNo}{0}
\NewDocumentCommand\EP{smmmm}%
{%
\stepcounter{EPNo}\roman{EPNo}. &%
\def\eL{.6cm}%
\IfStrEqCase{#2}%
{%
D{\gdef\eL{1cm}}%
E{\gdef\eL{.75cm}}%
F{\gdef\eL{.35cm}}%
G{\gdef\eL{.35cm}}%
}%
\IfBooleanTF{#1}%
{\dynkin[edge length=\eL,backwards,labels*={#4},labels={#5}]{#2}{#3}}
{\dynkin[edge length=\eL,labels*={#4},labels={#5}]{#2}{#3}}
\\
}%
\renewcommand*\do[1]{\EP{#1}}%
\begin{longtable}{MM}
\caption{Dynkin diagrams from Euler products \cite{Langlands:1967}}\\
\endfirsthead
\caption{\dots continued}\\
\endhead
\multicolumn{2}{c}{continued \dots}\\
\endfoot
\endlastfoot
\docsvlist{
A{***.}*{1,1,1,1,1}{1,2,n-1,n},
A{***.}*{1,1,1,1,1}{1,2,n-1,n},
A{**.***.*}{1,1,1,1,1,1}{1,2,m-1,,m,n},
B{**.***}{2,2,2,2,1}{1,2,n-1,n},
*B{***.}*{2,2,2,2,1}{n,n-1,2,1,},
C{**.***}{1,1,1,1,2}{1,2,n-1,},
*C{***.}*{1,1,1,1,2}{n,n-1,2,1,},
D{**.*****}{1,1,1,1,1,1}{1,2,n-2,n-1,n},
D{**.*****}{1,1,1,1,1,1}{1,2,n-2,n-1,n},
E6{1,1,1,1,1,1}{1,...,5},
*E7{1,1,1,1,1,1,1}{6,...,1},
E7{1,1,1,1,1,1,1}{1,...,6},

```



```

*E8{1,1,1,1,1,1,1,1}{7,...,1},
E8{1,1,1,1,1,1,1,1}{1,...,7},
G2{1,3}{1},
G2{1,3}{1},
B{**.*.**}{2,2,2,2,1}{1,2,n-1,n},
F4{1,1,2,2}{3,2,1},
C3{1,1,2}{2,1},
C{**.**}{1,1,1,2}{1,n-2,n-1,n},
*B3{2,2,1}{1,2},
F4{1,1,2,2}{1,2,3},
D{**.**}{1,1,1,1,1,1}{1,2,n-2,n-2,n,n},
E6{1,1,1,1,1,1}{1,2,3,4,,5},
E6{1,1,1,1,1,1}{1,2,3,5,,4},
*E7{1,1,1,1,1,1,1}{5,...,1,6},
*E7{1,1,1,1,1,1,1}{6,4,3,2,1,5},
*E8{1,1,1,1,1,1,1,1}{6,...,1,7},
*E8{1,1,1,1,1,1,1,1}{7,5,4,3,2,1,6},
*E7{1,1,1,1,1,1,1}{5,...,1,6},
*E7{1,1,1,1,1,1,1}{1,...,5,6},
*E8{1,1,1,1,1,1,1,1}{6,...,1,7}}
\end{longtable}

```

11. STYLE

Colours

```

\dynkin[
  edge/.style={blue!50,thick},
  */.style=blue!50!red,
  arrow style=red]{F}{4}

```



Arrow shapes

```

\dynkin[arrow shape/{-}{Stealth[blue,width=3mm]}]{F} F4

```



Edge lengths

The Dynkin diagram of (A_3) is `\dynkin[edge length=1.2]A3`

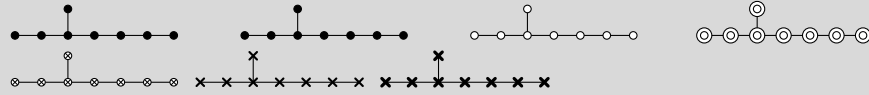
The Dynkin diagram of A_3 is

Root marks

```

\dynkin E8
\dynkin[mark=*]E8
\dynkin[mark=o]E8
\dynkin[mark=O]E8
\dynkin[mark=t]E8
\dynkin[mark=x]E8
\dynkin[mark=X]E8

```



At the moment, you can only use:

- * • solid dot
- o ○ hollow circle
- O ⊙ double hollow circle
- t ⊗ tensor root
- x × crossed root
- X × thickly crossed root

Mark styles

The parabolic subgroup $(E_{8,124})$ is
`\dynkin[parabolic=124,x/.style={brown,very thick}]E8`

The parabolic subgroup $E_{8,124}$ is

Sizes of root marks

$(A_{3,3})$ with big root marks is `\dynkin[root radius=.08cm,parabolic=3]A3`

$A_{3,3}$ with big root marks is

12. SUPPRESS OR REVERSE ARROWS

Some diagrams have double or triple edges

```
\dynkin F4
\dynkin G2
```



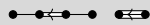
Suppress arrows

```
\dynkin[arrows=false]F4
\dynkin[arrows=false]G2
```



Reverse arrows

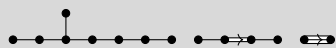
```
\dynkin[reverse arrows]F4
\dynkin[reverse arrows]G2
```



13. BACKWARDS AND UPSIDE DOWN

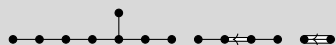
Default

```
\dynkin E8
\dynkin F4
\dynkin G2
```



Backwards

```
\dynkin[backwards]E8
\dynkin[backwards]F4
\dynkin[backwards]G2
```



Reverse arrows

```
\dynkin[reverse arrows]F4
\dynkin[reverse arrows]G2
```



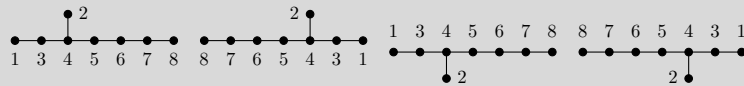
Backwards, reverse arrows

```
\dynkin[backwards,reverse arrows]F4
\dynkin[backwards,reverse arrows]G2
```



Backwards versus upside down

```
\dynkin[label]E8
\dynkin[label,backwards]E8
\dynkin[label,upside down]E8
\dynkin[label,backwards,upside down]E8
```



14. DRAWING ON TOP OF A DYNKIN DIAGRAM

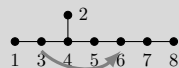
TikZ can access the roots themselves

```
\begin{dynkinDiagram}A4
  \fill[white,draw=black] (root 2) circle (.15cm);
  \fill[white,draw=black] (root 2) circle (.1cm);
  \draw[black] (root 2) circle (.05cm);
\end{dynkinDiagram}
```



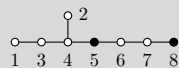
Draw curves between the roots

```
\begin{dynkinDiagram}[label]E8
  \draw[very thick, black!50,-latex]
    (root 3.south) to [out=-45, in=-135] (root 6.south);
\end{dynkinDiagram}
```



Change marks

```
\begin{dynkinDiagram}[mark=o,label]E8
  \dynkinRootMark{*}5
  \dynkinRootMark{*}8
\end{dynkinDiagram}
```



15. MARK LISTS

The package allows a list of root marks instead of a rank:

A mark list

```
\dynkin E{oo**ttxx}
```



The mark list `oo**ttxx` has one mark for each root: `o`, `o`, `*`, `*`, `t`, `t`, `x`, `x`. Roots are listed in the current default ordering. (Careful: in an affine root system, a mark list will *not* contain a mark for root zero.)

If you need to repeat a mark, you can give a *single digit* positive integer to indicate how many times to repeat it.

A mark list with repetitions

```
\dynkin A{x4o3t4}
```

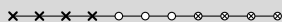


Table 5: Classical Lie superalgebras [10]. We need a slightly larger root radius parameter to distinguish the tensor product symbols from the solid dots.

		<code>\tikzset{/Dynkin diagram,root radius=.07cm}</code>
A_{mn}		<code>\dynkin A{o3.oto.oo}</code>
B_{mn}		<code>\dynkin B{o3.oto.oo}</code>
B_{0n}		<code>\dynkin B{o3.o3.o*}</code>
C_n		<code>\dynkin C{too.oto.oo}</code>
D_{mn}		<code>\dynkin D{o3.oto.o4}</code>
$D_{21\alpha}$		<code>\dynkin A{oto}</code>
F_4		<code>\dynkin F{ooot}</code>
G_3		<code>\dynkin[extended,affine mark=t, reverse arrows]G2</code>

Table 6: Classical Lie superalgebras [10]. Here we see the problem with using the default root radius parameter, which is too small for tensor product symbols.

A_{mn}		<code>\dynkin A{o3.oto.oo}</code>
B_{mn}		<code>\dynkin B{o3.oto.oo}</code>
B_{0n}		<code>\dynkin B{o3.o3.o*}</code>
C_n		<code>\dynkin C{too.oto.oo}</code>
D_{mn}		<code>\dynkin D{o3.oto.o4}</code>
$D_{21\alpha}$		<code>\dynkin A{oto}</code>
F_4		<code>\dynkin F{ooot}</code>
G_3		<code>\dynkin[extended,affine mark=t, reverse arrows]G2</code>

16. INDEFINITE EDGES

An *indefinite edge* is a dashed edge between two roots, $\bullet \cdots \bullet$ indicating that an indefinite number of roots have been omitted from the Dynkin diagram. In between any two entries in a mark list, place a period to indicate an indefinite edge:

Indefinite edges

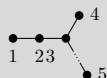
`\dynkin D{o.o*.t.to.t}`



In certain diagrams, roots may have an edge between them even though they are not subsequent in the ordering. For such rare situations, there is an option:

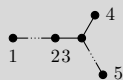
Indefinite edge option

```
\dynkin[make indefinite edge={3-5},label]D5
```



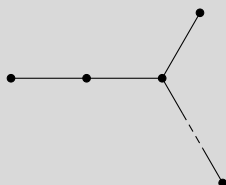
Give a list of edges to become indefinite

```
\dynkin[make indefinite edge/.list={1-2,3-5},label]D5
```



Indefinite edge style

```
\dynkin[indefinite edge/.style={
  draw=black,fill=white,thin,densely dashed},
  edge length=1cm,
  make indefinite edge={3-5}]D5
```



The ratio of the lengths of indefinite edges to those of other edges

```
\dynkin[edge length = .5cm,
  indefinite edge ratio=3,
  make indefinite edge={3-5}]D5
```

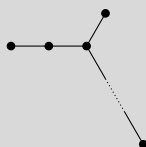


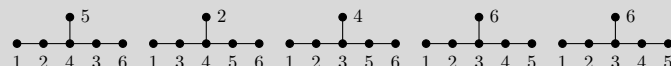
Table 7: Springer’s table of indices [24], pp. 320-321, with one form of E_7 corrected

A_n		
A_n		
B_n		
C_n		
D_n		
E_6		<code>\dynkin E{*oooo*}</code>
E_6		<code>\dynkin E{o*o*oo}</code>
E_6		<code>\dynkin E{o*oooo}</code>
E_6		<code>\dynkin E{**ooo*}</code>
E_7		<code>\dynkin E{*oooooo}</code>
E_7		<code>\dynkin E{oooooo*o}</code>
E_7		<code>\dynkin E{oooooo*}</code>
E_7		<code>\dynkin E{*oooo*o}</code>
E_7		<code>\dynkin E{*oooo**}</code>
E_7		<code>\dynkin E{*o**o*o}</code>
E_8		<code>\dynkin E{*ooooooo}</code>
E_8		<code>\dynkin E{ooooooo*}</code>
E_8		<code>\dynkin E{*oooooo*}</code>
E_8		<code>\dynkin E{oooooo**}</code>
E_8		<code>\dynkin E{*oooo***}</code>
F_4		<code>\dynkin F{ooo*}</code>
D_4		<code>\dynkin D{o*oo}</code>

17. ROOT ORDERING

Root ordering

```
\dynkin[label,ordering=Adams]E6
\dynkin[label,ordering=Bourbaki]E6
\dynkin[label,ordering=Carter]E6
\dynkin[label,ordering=Dynkin]E6
\dynkin[label,ordering=Kac]E6
```

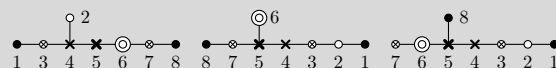


Default is Bourbaki. Sources are Adams [1] p. 56–57, Bourbaki [3] p. pp. 265–290 plates I–IX, Carter [5] p. 540–609, Dynkin [8], Kac [15] p. 43.

	Adams	Bourbaki	Carter	Dynkin	Kac
E_6					
E_7					
E_8					
F_4					
G_2					

The marks are set down in order according to the current root ordering:

```
\dynkin[label]E{*otxXOt*}
\dynkin[label,ordering=Carter]E{*otxXOt*}
\dynkin[label,ordering=Kac]E{*otxXOt*}
```



Convert between orderings

```
\newcount\r
\dynkinOrder E8.Carter::6->Bourbaki.{\r}
```

In (E_8) , root 6 in Carter's ordering is root $\text{\the\rr{}}$ in Bourbaki's ordering.

In E_8 , root 6 in Carter's ordering is root 2 in Bourbaki's ordering.

18. PARABOLIC SUBGROUPS

Each set of roots is assigned a number, with each binary digit zero or one to say whether the corresponding root is crossed or not:

The flag variety of pointed lines in projective 3-space is associated to the Dynkin diagram $\text{\dynkin[parabolic=3]A3}$.

The flag variety of pointed lines in projective 3-space is associated to the Dynkin diagram $\text{\dynkin[parabolic=3]A3}$.

Table 9: The Hermitian symmetric spaces

A_n		Grassmannian of k -planes in \mathbb{C}^{n+1}
B_n		$(2n - 1)$ -dimensional hyperquadric, i.e. the variety of null lines in \mathbb{C}^{2n+1}
C_n		space of Lagrangian n -planes in \mathbb{C}^{2n}
D_n		$(2n - 2)$ -dimensional hyperquadric, i.e. the variety of null lines in \mathbb{C}^{2n}
D_n		one component of the variety of maximal dimension null subspaces of \mathbb{C}^{2n}
D_n		the other component
E_6		complexified octave projective plane
E_6		its dual plane
E_7		the space of null octave 3-planes in octave 6-space

```

\NewDocumentCommand\HSS{mommm}
{#1&\IfNoValueTF{#2}{\dynkin{#3}{#4}}{\dynkin[parabolic=#2]{#3}{#4}}\&#5\\}
\renewcommand*{\arraystretch}{1.5}
\begin{longtable}
{>{\columncolor[gray]{.9}}>$1<$>{\columncolor[gray]{.9}}>$1<$>{\columncolor[gray]{.9}}1}
\caption{The Hermitian symmetric spaces}\endfirsthead
\caption{\dots continued}\endhead
\caption{continued \dots}\endfoot
\endlastfoot
\HSS{A_n}A{**.*x*.*}{Grassmannian of $k$-planes in $\mathbb{C}^{n+1}$}
\HSS{B_n}[1]B{\$(2n-1)$-dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n+1}$}
\HSS{C_n}[16]C{\space of Lagrangian $n$-planes in $\mathbb{C}^{2n}$}

```

```

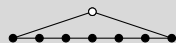
\HSS{D_n}[1]D{\$(2n-2)$-dimensional hyperquadric, i.e. the variety of null lines in $\mathbb{C}^{2n}$}
\HSS{D_n}[32]D{\{one component of the variety of maximal dimension null subspaces of $\mathbb{C}^{2n}$\}}
\HSS{D_n}[16]D{\{the other component\}}
\HSS{E_6}[1]E6{\text{complexified octave projective plane}}
\HSS{E_6}[32]E6{\text{its dual plane}}
\HSS{E_7}[64]E7{\text{the space of null octave 3-planes in octave 6-space}}
\end{longtable}

```

19. EXTENDED DYNKIN DIAGRAMS

Extended Dynkin diagrams

```
\dynkin[extended]A7
```



The extended Dynkin diagrams are also described in the notation of Kac [15] p. 55 as affine untwisted Dynkin diagrams: we extend `\dynkin{A}{7}` to become `\dynkin A[1]7`:

Extended Dynkin diagrams

```
\dynkin A[1]7
```

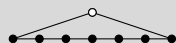


Table 10: The Dynkin diagrams of the extended simple root systems

A_1^1		<code>\dynkin[extended]A1</code>
A_n^1		<code>\dynkin[extended]A{}</code>
B_n^1		<code>\dynkin[extended]B{}</code>
C_n^1		<code>\dynkin[extended]C{}</code>
D_n^1		<code>\dynkin[extended]D{}</code>
E_6^1		<code>\dynkin[extended]E6</code>
E_7^1		<code>\dynkin[extended]E7</code>
E_8^1		<code>\dynkin[extended]E8</code>
F_4^1		<code>\dynkin[extended]F4</code>
G_2^1		<code>\dynkin[extended]G2</code>

20. AFFINE TWISTED AND UNTWISTED DYNKIN DIAGRAMS

The affine Dynkin diagrams are described in the notation of Kac [15] p. 55:

Affine Dynkin diagrams

$\backslash(A^{\{1\}}_7=\backslash\text{dynkin A}[1]7, \backslash$
 $E^{\{2\}}_6=\backslash\text{dynkin E}[2]6, \backslash$
 $D^{\{3\}}_4=\backslash\text{dynkin D}[3]4\backslash$

$$A_7^{(1)} = \text{diagram}, E_6^{(2)} = \text{diagram}, D_4^{(3)} = \text{diagram}$$

Table 11: The affine Dynkin diagrams

A_1^1		$\backslash\text{dynkin A}[1]1$
A_n^1		$\backslash\text{dynkin A}[1]\{\}$
B_n^1		$\backslash\text{dynkin B}[1]\{\}$
C_n^1		$\backslash\text{dynkin C}[1]\{\}$
D_n^1		$\backslash\text{dynkin D}[1]\{\}$
E_6^1		$\backslash\text{dynkin E}[1]6$
E_7^1		$\backslash\text{dynkin E}[1]7$
E_8^1		$\backslash\text{dynkin E}[1]8$
F_4^1		$\backslash\text{dynkin F}[1]4$
G_2^1		$\backslash\text{dynkin G}[1]2$
A_2^2		$\backslash\text{dynkin A}[2]2$
A_{ev}^2		$\backslash\text{dynkin A}[2]\{\text{even}\}$
A_{od}^2		$\backslash\text{dynkin A}[2]\{\text{odd}\}$
D_n^2		$\backslash\text{dynkin D}[2]\{\}$
E_6^2		$\backslash\text{dynkin E}[2]6$
D_4^3		$\backslash\text{dynkin D}[3]4$

Table 12: Some more affine Dynkin diagrams

$$A_4^2 \quad \text{diagram} \quad \backslash\text{dynkin A}[2]4$$

continued ...

Table 12: ...continued

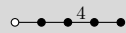
A_5^2		<code>\dynkin A[2]5</code>
A_6^2		<code>\dynkin A[2]6</code>
A_7^2		<code>\dynkin A[2]7</code>
A_8^2		<code>\dynkin A[2]8</code>
D_3^2		<code>\dynkin D[2]3</code>
D_4^2		<code>\dynkin D[2]4</code>
D_5^2		<code>\dynkin D[2]5</code>
D_6^2		<code>\dynkin D[2]6</code>
D_7^2		<code>\dynkin D[2]7</code>
D_8^2		<code>\dynkin D[2]8</code>
D_4^3		<code>\dynkin D[3]4</code>
E_6^2		<code>\dynkin E[2]6</code>

Table 13: Some more Kac–Moody Dynkin diagrams, only allowed in Kac ordering

E_6		<code>\dynkin[ordering=Kac,label]E6</code>
E_7		<code>\dynkin[ordering=Kac,label]E7</code>
E_8		<code>\dynkin[ordering=Kac,label]E8</code>
E_9		<code>\dynkin[ordering=Kac,label]E9</code>
E_{10}		<code>\dynkin[ordering=Kac,label]E{10}</code>
E_{11}		<code>\dynkin[ordering=Kac,label]E{11}</code>

21. EXTENDED COXETER DIAGRAMS

Extended and Coxeter options together

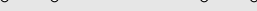
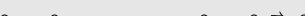
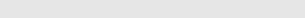

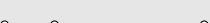
`\dynkin[extended,Coxeter]F4`

A_n		<code>\dynkin[extended,Coxeter]A{}</code>
B_n		<code>\dynkin[extended,Coxeter]B{}</code>
C_n		<code>\dynkin[extended,Coxeter]C{}</code>
D_n		<code>\dynkin[extended,Coxeter]D{}</code>
E_6		<code>\dynkin[extended,Coxeter]E6</code>
E_7		<code>\dynkin[extended,Coxeter]E7</code>
E_8		<code>\dynkin[extended,Coxeter]E8</code>
F_4		<code>\dynkin[extended,Coxeter]F4</code>
G_2		<code>\dynkin[extended,Coxeter]G2</code>
H_3		<code>\dynkin[extended,Coxeter]H3</code>
H_4		<code>\dynkin[extended,Coxeter]H4</code>
I_1		<code>\dynkin[extended,Coxeter]I1</code>

We include a style called `Kac` which tries to imitate the style of [15].

 $\backslash\text{dynkin[Kac]F4}$

○ — ○ \Rightarrow ○ — ○

A_n		<code>\dynkin A{}</code>
B_n		<code>\dynkin B{}</code>
C_n		<code>\dynkin C{}</code>
D_n		<code>\dynkin D{}</code>
E_6		<code>\dynkin E6</code>

continued ...

Table 15: ...continued

E_7		<code>\dynkin E7</code>
E_8		<code>\dynkin E8</code>
F_4	$\circ - \circ \Rightarrow \circ - \circ$	<code>\dynkin F4</code>
G_2	$\circ \Rightarrow \circ$	<code>\dynkin G2</code>

Table 16: The Dynkin diagrams of the extended simple root systems in Kac style

A_1^1	$\Leftarrow \Rightarrow \circ$	<code>\dynkin[extended]A1</code>
A_n^1		<code>\dynkin[extended]A{}</code>
B_n^1		<code>\dynkin[extended]B{}</code>
C_n^1	$\circ \Rightarrow \circ - \circ - \dots - \circ - \circ \Leftarrow \circ$	<code>\dynkin[extended]C{}</code>
D_n^1		<code>\dynkin[extended]D{}</code>
E_6^1		<code>\dynkin[extended]E6</code>
E_7^1		<code>\dynkin[extended]E7</code>
E_8^1		<code>\dynkin[extended]E8</code>
F_4^1	$\circ - \circ - \circ \Rightarrow \circ - \circ$	<code>\dynkin[extended]F4</code>
G_2^1	$\circ - \circ \Rightarrow \circ$	<code>\dynkin[extended]G2</code>

Table 17: The Dynkin diagrams of the twisted simple root systems in Kac style

A_2^2	$\circ \Leftarrow \circ$	<code>\dynkin A[2]2</code>
A_{ev}^2	$\circ \Leftarrow \circ - \circ - \circ - \cdots - \circ - \circ \Leftarrow \circ$	<code>\dynkin A[2]{even}</code>
A_{od}^2	$\begin{array}{c} \circ \\ \diagdown \\ \circ - \circ - \circ - \cdots - \circ - \circ \Leftarrow \circ \\ \diagup \\ \circ \end{array}$	<code>\dynkin A[2]{odd}</code>
D_n^2	$\circ \Leftarrow \circ - \circ - \cdots - \circ - \circ \Rightarrow \circ$	<code>\dynkin D[2]{}</code>
E_6^2	$\circ - \circ - \circ \Leftarrow \circ - \circ$	<code>\dynkin E[2]6</code>
D_4^3	$\circ - \circ \Leftarrow \circ$	<code>\dynkin D[3]4</code>

23. CEREF STYLE

We include a style called `ceref` which paints oblong root markers with shadows. The word “ceref” is an old form of the word “serif”.

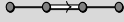
Ceref style
<code>\dynkin[ceref]F4</code>


Table 18: The Dynkin diagrams of the simple root systems in ceref style

A_n	$\bullet - \bullet - \cdots - \bullet$	<code>\dynkin A{}</code>
B_n	$\bullet - \bullet - \cdots - \bullet \rightrightarrows \bullet$	<code>\dynkin B{}</code>
C_n	$\bullet - \bullet - \cdots - \bullet \Leftarrow \bullet$	<code>\dynkin C{}</code>
D_n	$\bullet - \bullet - \cdots - \bullet \begin{array}{l} \nearrow \bullet \\ \searrow \bullet \end{array}$	<code>\dynkin D{}</code>
E_6	$\begin{array}{c} \bullet \\ \\ \bullet - \bullet - \bullet - \bullet - \bullet \end{array}$	<code>\dynkin E6</code>
E_7	$\begin{array}{c} \bullet \\ \\ \bullet - \bullet - \bullet - \bullet - \bullet - \bullet \end{array}$	<code>\dynkin E7</code>
E_8	$\begin{array}{c} \bullet \\ \\ \bullet - \bullet - \bullet - \bullet - \bullet - \bullet - \bullet \end{array}$	<code>\dynkin E8</code>
F_4	$\bullet - \bullet \rightrightarrows \bullet - \bullet$	<code>\dynkin F4</code>
G_2	$\bullet \rightrightarrows \bullet$	<code>\dynkin G2</code>

Table 19: The Dynkin diagrams of the extended simple root systems in cref style

A_1^1		<code>\dynkin[extended]A1</code>
A_n^1		<code>\dynkin[extended]A{}</code>
B_n^1		<code>\dynkin[extended]B{}</code>
C_n^1		<code>\dynkin[extended]C{}</code>
D_n^1		<code>\dynkin[extended]D{}</code>
E_6^1		<code>\dynkin[extended]E6</code>
E_7^1		<code>\dynkin[extended]E7</code>
E_8^1		<code>\dynkin[extended]E8</code>
F_4^1		<code>\dynkin[extended]F4</code>
G_2^1		<code>\dynkin[extended]G2</code>

Table 20: The Dynkin diagrams of the twisted simple root systems in cref style

A_2^2		<code>\dynkin A[2]2</code>
A_{ev}^2		<code>\dynkin A[2]{even}</code>
A_{od}^2		<code>\dynkin A[2]{odd}</code>
D_n^2		<code>\dynkin D[2]{}</code>
E_6^2		<code>\dynkin E[2]6</code>
D_4^3		<code>\dynkin D[3]4</code>

24. MORE ON FOLDED DYNKIN DIAGRAMS

The Dynkin diagrams package has limited support for folding Dynkin diagrams.

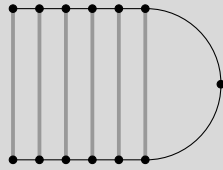
Folding

`\dynkin[fold]A{13}`



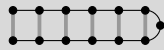
Big fold radius

```
\dynkin[fold,fold radius=1cm]A{13}
```



Small fold radius

```
\dynkin[fold,fold radius=.2cm]A{13}
```



Some Dynkin diagrams have multiple foldings, which we attempt to distinguish (not entirely successfully) by their *ply*: the maximum number of roots folded together. Most diagrams can only allow a 2-ply folding, so fold is a synonym for ply=2.

3-ply

```
\dynkin[ply=3]D4  
\dynkin[ply=3,fold right]D4  
\dynkin[ply=3]D[1]4
```



4-ply

```
\dynkin[ply=4]D[1]4
```



The $D_\ell^{(1)}$ diagrams can be folded on their left end and separately on their right end:

Left, right and both

```
\dynkin D[1]{ } \
\dynkin[fold left]D[1]{ } \
\dynkin[fold right]D[1]{ } \
\dynkin[fold]D[1]{ }
```



We have to be careful about the 4-ply foldings of $D_{2\ell}^{(1)}$, for which we can have two different patterns, so by default, the package only draws as much as it can without distinguishing the two:

Default $D_{2\ell}^{(1)}$ and the two ways to finish it

```
\dynkin[ply=4]D[1]{****.*****.*****}%
\
\begin{dynkinDiagram}[ply=4]{D}[1]{****.*****.*****}%
\dynkinFold[bend right=90]1{13}%
\dynkinFold[bend right=90]0{14}%
\end{dynkinDiagram} \
\begin{dynkinDiagram}[ply=4]{D}[1]{****.*****.*****}%
\dynkinFold01%
\dynkinFold1{13}%
\dynkinFold{13}{14}%
\end{dynkinDiagram}
```

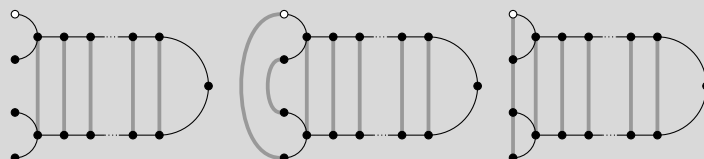


Table 21: Some foldings of Dynkin diagrams. For these diagrams, we want to compare a folding diagram with the diagram that results when we fold it, so it looks best to set `fold radius` and `edge length` to equal lengths.

A_3		<code>\dynkin[fold]A[0]3</code>
C_2		<code>\dynkin C[0]2</code>

continued ...

Table 21: ...continued

$A_{2\ell-1}$		<code>\dynkin[fold]A{**.****.**}</code>
C_ℓ		<code>\dynkin C{}</code>
B_3		<code>\dynkin[fold]B[0]3</code>
G_2		<code>\dynkin[reverse arrows]G[0]2</code>
D_4		<code>\dynkin[ply=3,fold right]D4</code>
G_2		<code>\dynkin G2</code>
$D_{\ell+1}$		<code>\dynkin[fold]D{}</code>
B_ℓ		<code>\dynkin B{}</code>
E_6		<code>\dynkin[fold]E[0]6</code>
F_4		<code>\dynkin[reverse arrows]F[0]4</code>
A_3^1		<code>\dynkin[ply=4]A[1]3</code>
A_1^1		<code>\dynkin A[1]1</code>
$A_{2\ell-1}^1$		<code>\dynkin[fold]A[1]{**.****.**}</code>
C_ℓ^1		<code>\dynkin C[1]{}</code>
B_3^1		<code>\dynkin[ply=3]B[1]3</code>
A_2^2		<code>\dynkin A[2]2</code>
B_3^1		<code>\dynkin[ply=2]B[1]3</code>
G_2^1		<code>\dynkin G[1]2</code>
B_ℓ^1		<code>\dynkin[fold]B[1]{}</code>
D_ℓ^2		<code>\dynkin D[2]{}</code>
D_4^1		<code>\dynkin[ply=3]D[1]4</code>
B_3^1		<code>\dynkin B[1]3</code>

continued ...

Table 21: ...continued

D_4^1		<code>\dynkin[ply=3]D[1]4</code>
G_2^1		<code>\dynkin G[1]2</code>
$D_{\ell+1}^1$		<code>\dynkin[fold]D[1]{}</code>
D_ℓ^2		<code>\dynkin D[2]{}</code>
$D_{\ell+1}^1$		<code>\dynkin[fold right]D[1]{}</code>
B_ℓ^1		<code>\dynkin B[1]{}</code>
$D_{2\ell}^1$		<pre> \begin{dynkinDiagram}[ply=4]D[1]% {****.*****.*****} \dynkinFold01 \dynkinFold1{13} \dynkinFold{13}{14} \end{dynkinDiagram} </pre>
A_{odd}^2		<code>\dynkin A[2]{odd}</code>
$D_{2\ell}^1$		<pre> \begin{dynkinDiagram}[ply=4]{D}[1]% {****.*****.*****} \dynkinFold[bend right=90]1{13} \dynkinFold[bend right=90]0{14} \end{dynkinDiagram} </pre>
A_{even}^2		<code>\dynkin A[2]{even}</code>
E_6^1		<code>\dynkin[fold]E[1]6</code>
F_4^1		<code>\dynkin[reverse arrows]F[1]4</code>
E_6^1		<code>\dynkin[ply=3]E[1]6</code>
D_4^3		<code>\dynkin D[3]4</code>
E_7^1		<code>\dynkin[fold]E[1]7</code>
E_6^2		<code>\dynkin E[2]6</code>

continued ...

Table 21: ...continued

F_4^1		<code>\dynkin[fold]F[1]4</code>
G_2^1		<code>\dynkin G[1]2</code>
A_{odd}^2		<code>\dynkin[odd, fold]A[2]{****.***}</code>
A_{even}^2		<code>\dynkin A[2]{even}</code>
D_3^2		<code>\dynkin[fold]D[2]3</code>
A_2^2		<code>\dynkin A[2]2</code>

Table 22: Frobenius fixed point subgroups of finite simple groups of Lie type [4] p. 15

$A_{\ell \geq 1}$		<code>\dynkin A{}</code>
${}^2A_{\ell \geq 2}$		<code>\dynkin[fold]A{}</code>
$B_{\ell \geq 2}$		<code>\dynkin B{}</code>
2B_2		<code>\dynkin[fold]B2</code>
$C_{\ell \geq 3}$		<code>\dynkin C{}</code>
$D_{\ell \geq 4}$		<code>\dynkin D{}</code>
${}^2D_{\ell \geq 4}$		<code>\dynkin[fold]D{}</code>
3D_4		<code>\dynkin[ply=3]D4</code>
E_6		<code>\dynkin E6</code>
2E_6		<code>\dynkin[fold]E6</code>
E_7		<code>\dynkin E7</code>
E_8		<code>\dynkin E8</code>
F_4		<code>\dynkin F4</code>
2F_4		<code>\dynkin[fold]F4</code>
G_2		<code>\dynkin G2</code>
2G_2		<code>\dynkin[fold]G2</code>

25. TYPESETTING MATHEMATICAL NAMES OF DYNKIN DIAGRAMS

The `\dynkinName` command, with the same syntax as `\dynkin`, typesets a default name of your diagram in \LaTeX . It is perhaps only useful when automatically generating a large collection of Dynkin diagrams in a computer program.

Name of a diagram

```
\dynkinName[label,extended]B7
\dynkinName A[2]{even}
\dynkinName[Coxeter]B7
\dynkinName[label,extended]B*
\dynkinName D[3]4
```

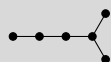
B_7^1 A_{ev}^2 B_7 B_1^1 D_4^3

26. CONNECTING DYNKIN DIAGRAMS

We can make some sophisticated folded diagrams by drawing multiple diagrams, each with a name:

Name a diagram

```
\dynkin[name=Bob]D6
```



We can then connect the two with folding edges:

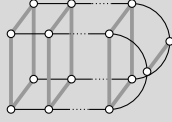
Connect diagrams

```
\begin{dynkinDiagram}[name=upper]A3
  \node (current) at ($(\text{upper root 1})+(0,-.3\text{cm})$) {};
  \dynkin[at=(current),name=lower]A3
  \begin{pgfonlayer}{Dynkin behind}
    \foreach \i in {1,...,3}%
    {%
      \draw[/Dynkin diagram/fold style]
        ($(\text{upper root } \i)$)
        -- ($(\text{lower root } \i)$);%
    }%
  \end{pgfonlayer}
\end{dynkinDiagram}
```

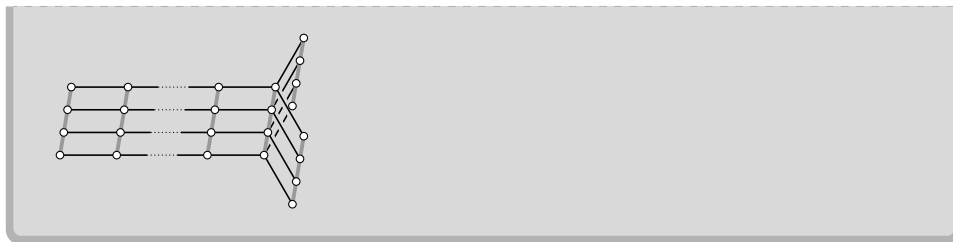
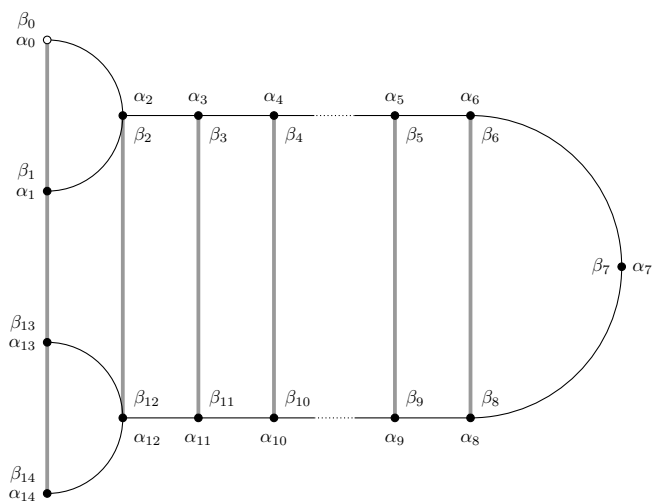
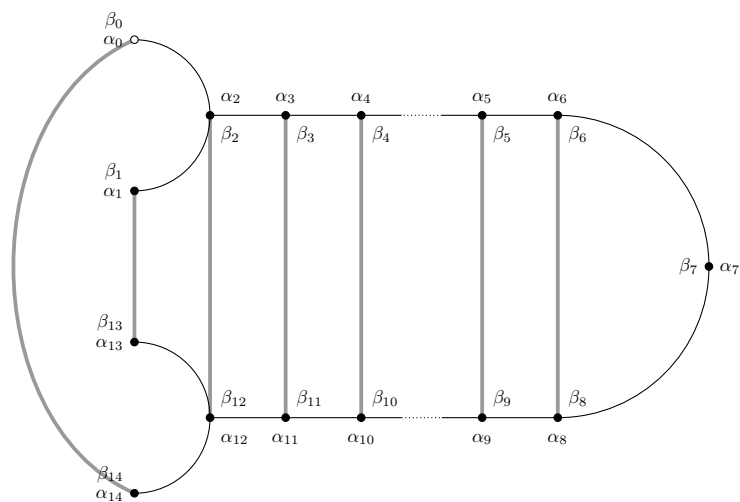


The following diagrams arise in the Satake diagrams of the pseudo-Riemannian symmetric spaces [2].

```
\pgfkeys{/Dynkin diagram,edge length=.5cm,fold radius=.5cm}
\begin{tikzpicture}
  \dynkin[name=1]A{IIIb}
  \node (a) at (-.3,-.4){};
  \dynkin[name=2,at=(a)]A{IIIb}
  \begin{pgfonlayer}{Dynkin behind}
    \foreach \i in {1,...,7}%
    {%
      \draw[/Dynkin diagram/fold style]
        ($ (1 root \i)$)
        --
        ($ (2 root \i)$);%
    }%
  \end{pgfonlayer}
\end{tikzpicture}
```



```
\pgfkeys{/Dynkin diagram,
edge length=.75cm,
edge/.style={draw=example-color,double=black,very thick}}
\begin{tikzpicture}
  \foreach \d in {1,...,4}
  {
    \node (current) at ($ (\d*.05,\d*.3)$) {};
    \dynkin[name=\d,at=(current)]D{oo.oooo}
  }
  \begin{pgfonlayer}{Dynkin behind}
    \foreach \i in {1,...,6}%
    {%
      \draw[/Dynkin diagram/fold style] ($ (1 root \i)$) -- ($ (2
root \i)$);%
      \draw[/Dynkin diagram/fold style] ($ (2 root \i)$) -- ($ (3
root \i)$);%
      \draw[/Dynkin diagram/fold style] ($ (3 root \i)$) -- ($ (4
root \i)$);%
    }%
  \end{pgfonlayer}
\end{tikzpicture}
```


¹ D_4 4-ply tied straight:¹ D_4 4-ply tied bending:

```

\tikzset{/Dynkin diagram,edge length=1cm,fold radius=1cm}
\tikzset{/Dynkin diagram,label macro/.code={\alpha_{#1}},label macro*/.code={\beta_{#1}}}
\({}^{-1}D_4\) 4-ply tied straight:
\begin{dynkinDiagram}[ply=4]D[1]%
{****.*****}
\dynkinFold 01
\dynkinFold 1{13}

```

```

\dynkinFold{13}{14}
\dynkinLabelRoots{0,...,14}
\dynkinLabelRoots*{0,...,14}
\end{dynkinDiagram}
\({}^1D_4\) 4-ply tied bending:
\begin{dynkinDiagram}[ply=4]D[1]%
{****.****.****}
\dynkinFold1{13}
\dynkinFold[bend right=65]0{14}
\dynkinLabelRoots{0,...,14}
\dynkinLabelRoots*{0,...,14}
\end{dynkinDiagram}

```

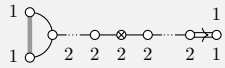
Below we draw the Vogan diagrams of some affine Lie superalgebras [21, 20].

$\mathfrak{sl}(2m|2n)^{(2)}$

```

\begin{dynkinDiagram}[ply=2,label]{B}[1]{oo.oto.oo}
\dynkinLabelRoot*71
\end{dynkinDiagram}

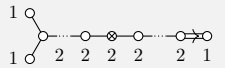
```



```

\dynkin[label]B[1]{oo.oto.oo}

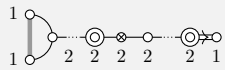
```



```

\dynkin[ply=2,label]B[1]{oo.Oto.Oo}

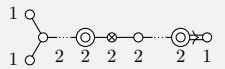
```



```

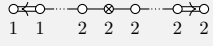
\dynkin[label]B[1]{oo.Oto.Oo}

```

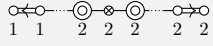


$$\mathfrak{sl}(2m+1|2n+1)^2$$

`\dynkin[label]D[2]{o.oto.oo}`

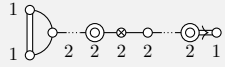


`\dynkin[label]D[2]{o.OtO.oo}`

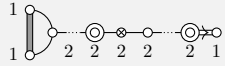


$$\mathfrak{sl}(2|2n+1)^{(2)}$$

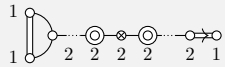
`\dynkin[ply=2,label,double edges]B[1]{oo.Oto.Oo}`



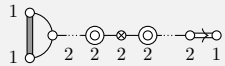
`\dynkin[ply=2,label,double fold]B[1]{oo.Oto.Oo}`

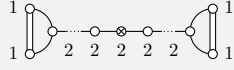
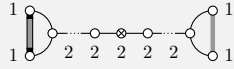
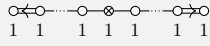
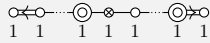
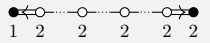


`\dynkin[ply=2,label,double edges]B[1]{oo.OtO.oo}`

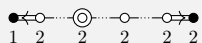


`\dynkin[ply=2,label,double fold]B[1]{oo.OtO.oo}`

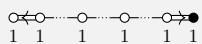


$\mathfrak{sl}(2|2n)^{(2)}$
`\dynkin[ply=2,label,double edges]D[1]{oo.oto.ooo}`

`\dynkin[ply=2,label,double fold left]D[1]{oo.oto.ooo}`

 $\mathfrak{osp}(2m|2n)^{(2)}$
`\dynkin[label,label macro/.code={1}]D[2]{o.oto.oo}`

`\dynkin[label,label macro/.code={1}]D[2]{o.Oto.Oo}`

 $\mathfrak{osp}(2|2n)^{(2)}$
`\dynkin[label,label macro/.code=\lablIt{#1},
affine mark=*]
D[2]{o.o.o.o*}`


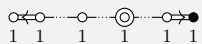
```
\dynkin[label,label macro/.code=\lablIt{#1},
  affine mark=*]
D[2]{o.O.o.o*}
```


 $\mathfrak{sl}(1|2n+1)^4$

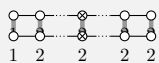
```
\dynkin[label,label macro/.code={1}]D[2]{o.o.o.o*}
```

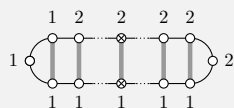
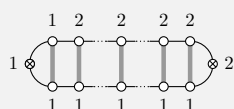
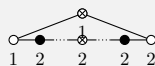
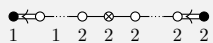
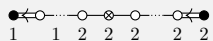
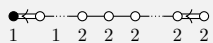


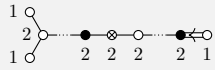
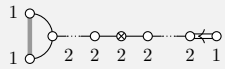
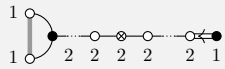
```
\dynkin[label,label macro/.code={1}]D[2]{o.o.O.o*}
```


 A^1

```
\begin{tikzpicture}
  \dynkin[name=upper]A{oo.t.oo}
  \node (Dynkin current) at (upper root 1){};
  \dynkinSouth
  \dynkin[at=(Dynkin current),name=lower]A{oo.t.oo}
  \begin{pgfonlayer}{Dynkin behind}
    \foreach \i in {1,...,5}{
      \draw[/Dynkin diagram/fold style]
        ($(\text{upper root \i})$) -- ($(\text{lower root \i})$);
    }
  \end{pgfonlayer}
\end{tikzpicture}
```



$\backslash\text{dynkin}[\text{fold}]A[1]\{\text{oo.t.oooo.t.oo}\}$

 $\backslash\text{dynkin}[\text{fold},\text{affine mark}=\text{t}]A[1]\{\text{oo.o.ootoo.o.oo}\}$

 $\backslash\text{dynkin}[\text{affine mark}=\text{t}]A[1]\{\text{o*.t.*o}\}$

 B^1
 $\backslash\text{dynkin}[\text{affine mark}=\text{*}]A[2]\{\text{o.oto.o*}\}$

 $\backslash\text{dynkin}[\text{affine mark}=\text{*}]A[2]\{\text{o.oto.o*}\}$

 $\backslash\text{dynkin}[\text{affine mark}=\text{*}]A[2]\{\text{o.ooo.oo}\}$


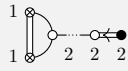
$\backslash\text{dynkin}[\text{odd}]A[2]\{\text{oo}.*\text{to}.*\text{o}\}$

 $\backslash\text{dynkin}[\text{odd},\text{fold}]A[2]\{\text{oo}.\text{oto}.\text{oo}\}$

 $\backslash\text{dynkin}[\text{odd},\text{fold}]A[2]\{\text{o}.*\text{oto}.\text{o}.*\}$

 D^1
 $\backslash\text{dynkin } D\{\text{otoo}\}$

 $\backslash\text{dynkin } D\{\text{ot}.*\text{o}\}$

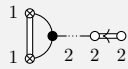
 $\backslash\text{dynkin}[\text{fold}]D\{\text{otoo}\}$


C^1

```
\dynkin[double edges,fold,affine mark=t,odd]A[2]{to.o*}
```



```
\dynkin[double edges,fold,affine mark=t,odd]A[2]{t*.oo}
```

 F^1

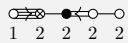
```
\begin{dynkinDiagram}A{oto*}%
  \dynkinQuadrupleEdge 12%
  \dynkinTripleEdge 43%
\end{dynkinDiagram}%
```



```
\begin{dynkinDiagram}A{*too}%
  \dynkinQuadrupleEdge 12%
  \dynkinTripleEdge 43%
\end{dynkinDiagram}%
```

 G^1

```
\begin{dynkinDiagram}A{ot*oo}%
  \dynkinQuadrupleEdge 12%
  \dynkinDefiniteDoubleEdge 43%
\end{dynkinDiagram}%
```



<pre> \begin{dynkinDiagram}A{oto*o}% \dynkinQuadrupleEdge 12% \dynkinDefiniteDoubleEdge 43% \end{dynkinDiagram}% </pre>	
<pre> \begin{dynkinDiagram}A{*too*}% \dynkinQuadrupleEdge 12% \dynkinDefiniteDoubleEdge 43% \end{dynkinDiagram}% </pre>	
<pre> \begin{dynkinDiagram}A{*tooo}% \dynkinQuadrupleEdge 12% \dynkinDefiniteDoubleEdge 43% \end{dynkinDiagram}% </pre>	

28. EXAMPLE: THE COMPLEX SIMPLE LIE ALGEBRAS

\mathfrak{g}	Diagram	Weights	Roots	Simple roots
A_n		$\frac{1}{n+1}\mathbb{Z}^{n+1} / \langle \sum e_j \rangle$	$e_i - e_j$	$e_i - e_{i+1}$
B_n		$\frac{1}{2}\mathbb{Z}^n$	$\pm e_i, \pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, e_n$
C_n		\mathbb{Z}^n	$\pm 2e_i, \pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, 2e_n$
D_n		$\frac{1}{2}\mathbb{Z}^n$	$\pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, \quad i \leq n-1$ $e_{n-1} + e_n$
E_8		$\frac{1}{2}\mathbb{Z}^8$	$\pm 2e_i \pm 2e_j, \quad i \neq j,$ $\sum_i (-1)^{m_i} e_i, \quad \sum m_i \text{ even}$	$2e_1 - 2e_2,$ $2e_2 - 2e_3,$ $2e_3 - 2e_4,$ $2e_4 - 2e_5,$ $2e_5 - 2e_6,$ $2e_6 + 2e_7,$ $-\sum e_j,$ $2e_6 - 2e_7$

\mathfrak{g}	Diagram	Weights	Roots	Simple roots
E_7		$\frac{1}{2}\mathbb{Z}^8 / \langle e_1 - e_2 \rangle$	quotient of E_8	quotient of E_8
E_6		$\frac{1}{3}\mathbb{Z}^8 / \langle e_1 - e_2, e_2 - e_3 \rangle$	quotient of E_8	quotient of E_8
F_4		\mathbb{Z}^4	$\pm 2e_i,$ $\pm 2e_i \pm 2e_j, \quad i \neq j,$ $\pm e_1 \pm e_2 \pm e_3 \pm e_4$	$2e_2 - 2e_3,$ $2e_3 - 2e_4,$ $2e_4,$ $e_1 - e_2 - e_3 - e_4$
G_2		$\mathbb{Z}^3 / \langle \sum e_j \rangle$	$\pm(1, -1, 0),$ $\pm(-1, 0, 1),$ $\pm(0, -1, 1),$ $\pm(2, -1, -1),$ $\pm(1, -2, 1),$ $\pm(-1, -1, 2)$	$(-1, 0, 1),$ $(2, -1, -1)$

```

\NewDocumentEnvironment{bunch}{}%
{\renewcommand*{\arraystretch}{1}\begin{array}{@{}l@{}}\midrule{\midrule\end{array}}
\small
\NewDocumentCommand\nc{mm}{\newcolumntype{#1}{>{\columncolor[gray]{.9}}>{\$}m{#2cm}<{\$}}
\nc{G}{.3}\nc{D}{2.1}\nc{W}{3}\nc{R}{3.7}\nc{S}{3}
\NewDocumentCommand\LieG{}{\mathfrak{g}}
\NewDocumentCommand\W{om}{\ensuremath{\mathbb{Z}^{\#2}\IfValueT{#1}{/\left<#1\right>}}}
\renewcommand*{\arraystretch}{1.5}
\NewDocumentCommand\quo{}{\text{quotient of } E_8}
\begin{longtable}{@{}GDWRS@{}}
\LieG&\text{Diagram}&\text{Weights}&\text{Roots}&\text{Simple roots}\\ \midrule\endfirsthead
\LieG&\text{Diagram}&\text{Weights}&\text{Roots}&\text{Simple roots}\\ \midrule\endhead
A_n&\dynkin A{}&\frac{1}{n+1}W[\sum e_j]{n+1}&e_i-e_j&e_i-e_{i+1}\\
B_n&\dynkin B{}&\frac{1}{2}W[n&\pm e_i, \pm e_i \pm e_j, i\ne j&e_i-e_{i+1}, e_n\\
C_n&\dynkin C{}&W[n&\pm 2 e_i, \pm e_i \pm e_j, i\ne j&e_i-e_{i+1}, 2e_n\\
D_n&\dynkin D{}&\frac{1}{2}W[n&\pm e_i \pm e_j, i\ne j &
\begin{bunch}e_i-e_{i+1},&i\le n-1\\e_{n-1}+e_n\end{bunch}\\
E_8&\dynkin E8&\frac{1}{2}W[8&
\begin{bunch}\pm 2e_i \pm 2e_j,&i\ne j,\\ \sum_{i=1}^m e_i,&\sum m_i \text{ even}\end{bunch}&
\begin{bunch}2e_1-2e_2,\\2e_2-2e_3,\\2e_3-2e_4,\\2e_4-2e_5,\\2e_5-2e_6,\\2e_6+2e_7,\\
-\sum e_j,\\2e_6-2e_7\end{bunch}\\
E_7&\dynkin E7&\frac{1}{2}W[e_1-e_2]8&\quo&\\
E_6&\dynkin E6&\frac{1}{3}W[e_1-e_2,e_2-e_3]8&\quo&\quo\\
F_4&\dynkin F4&W4&
\begin{bunch}\pm 2e_i,\\ \pm 2e_i \pm 2e_j, \quad i \ne j,\\ \pm e_1 \pm e_2 \pm e_3 \pm e_4\end{bunch}&
\begin{bunch}2e_2-2e_3,\\2e_3-2e_4,\\2e_4,\\e_1-e_2-e_3-e_4\end{bunch}\\
G_2&\dynkin G2&W[\sum e_j]3&
\begin{bunch}\pm(1,-1,0),\\ \pm(-1,0,1),\\ \pm(0,-1,1),\\ \pm(2,-1,-1),\\ \pm(1,-2,1),\\ \pm(-1,-1,2)\end{bunch}

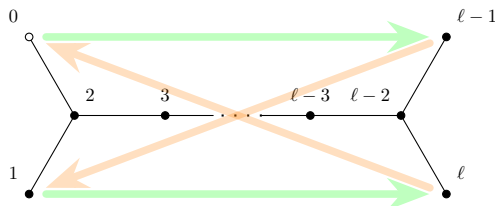
```

```

\end{bunch}&
\begin{bunch}(-1,0,1),\((2,-1,-1)\end{bunch}
\end{longtable}

```

29. AN EXAMPLE OF MIKHAIL BOROVoi



```

\tikzset{big arrow/.style={
-Stealth,line cap=round,line width=1mm,
shorten <=1mm,shorten >=1mm}}
\newcommand\catholic[2]{\draw[big arrow,green!25!white]
(root #1) to (root #2);}
\newcommand\protestant[2]{
\begin{scope}[transparency group, opacity=.25]
\draw[big arrow,orange] (root #1) to (root #2);
\end{scope}}
\begin{dynkinDiagram}[edge length=1.2cm,
indefinite edge/.style={thick,loosely dotted},
labels*={0,1,2,3,\ell-3,\ell-2,\ell-1,\ell}D[1]{
\catholic 06\catholic 17
\protestant 70\protestant 61
\end{dynkinDiagram}

```

30. SYNTAX

The syntax is `\dynkin[<options>]{<letter>[<twisted rank>]{<rank>}}` where `<letter>` is A, B, C, D, E, F or G, the family of root system for the Dynkin diagram, `<twisted rank>` is 0, 1, 2, 3 (default is 0) representing:

- 0 finite root system
- 1 affine extended root system, i.e. of type ⁽¹⁾
- 2 affine twisted root system of type ⁽²⁾
- 3 affine twisted root system of type ⁽³⁾

and `<rank>` is

- (1) an integer representing the rank or
- (2) blank to represent an indefinite rank or
- (3) the name of a Satake diagram as in section 4.

The environment syntax is `\begin{dynkinDiagram}` followed by the same parameters as `\dynkin`, then various Dynkin diagram and TikZ commands, and then `\end{dynkinDiagram}`.

31. OPTIONS

```

*/.style = <TikZ style data>,
default : solid,draw=black,fill=black
continued ...

```

Table 24: ...continued

style for roots like \bullet
`o/.style = <TikZ style data>,`
 default : `solid,draw=black,fill=white`
 style for roots like \circ
`O/.style = <TikZ style data>,`
 default : `solid,draw=black,fill=white`
 style for roots like \odot
`t/.style = <TikZ style data>,`
 default : `solid,draw=black,fill=black`
 style for roots like \otimes
`x/.style = <TikZ style data>,`
 default : `solid,draw=black,line cap=round`
 style for roots like \times
`X/.style = <TikZ style data>,`
 default : `solid,draw=black,thick,line cap=round`
 style for roots like \star
`affine mark = o,O,t,x,X,*,`
 default : `*`
 default root mark for root zero in an affine Dynkin diagram
`arrow shape/.style = TikZ style data,`
 default : `Computer Modern Rightarrow[black]`
 shape of arrow heads for most Dynkin diagrams that have arrows.
`arrow style = TikZ style data,`
 default : `black`
 set to override the default style for the arrows in nonsimply laced
 Dynkin diagrams, including length, width, line width and color
`arrows = <true or false>,`
 default : `true`
 whether to draw the arrows that arise along the edges.
`backwards = <true or false>,`
 default : `false`
 whether to reverse right to left.
`ceref = <true or false>,`
 default : `false`
 whether to draw roots in a “ceref” style.
`Coxeter = <true or false>,`
 default : `false`
 whether to draw a Coxeter diagram, rather than a Dynkin diagram.
`double edges = <>,`
 default : `not set`
 set to override the `fold` style when folding roots together in a
 Dynkin diagram, so that the foldings are indicated with double
 edges (like those of an F_4 Dynkin diagram without arrows).
`double fold = <>,`
 default : `not set`

continued ...

Table 24: ...continued

	set to override the <code>fold</code> style when folding roots together in a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows), but filled in solidly.
<code>double left = $\langle \rangle$,</code>	
<code>default : not set</code>	set to override the <code>fold</code> style when folding roots together at the left side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows).
<code>double fold left = $\langle \rangle$,</code>	
<code>default : not set</code>	set to override the <code>fold</code> style when folding roots together at the left side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows), but filled in solidly.
<code>double right = $\langle \rangle$,</code>	
<code>default : not set</code>	set to override the <code>fold</code> style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows).
<code>double fold right = $\langle \rangle$,</code>	
<code>default : not set</code>	set to override the <code>fold</code> style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an F_4 Dynkin diagram without arrows), but filled in solidly.
<code>edge label/.style = TikZ style data,</code>	
<code>default : text height=0,text depth=0,label distance=-2pt</code>	style of edge labels in the Dynkin diagram, as found, for example, on some Coxeter diagrams
<code>edge length = $\langle \text{number} \rangle$cm,</code>	
<code>default : .35cm</code>	distance between nodes in the Dynkin diagram
<code>edge/.style = TikZ style data,</code>	
<code>default : solid,draw=black,fill=white,thin</code>	style of edges in the Dynkin diagram
<code>fold = $\langle \text{true or false} \rangle$,</code>	
<code>default : true</code>	whether, when drawing Dynkin diagrams, to draw them 2-ply.
<code>fold left = $\langle \text{true or false} \rangle$,</code>	
<code>default : true</code>	whether to fold the roots on the left side of a Dynkin diagram.
<code>fold radius = $\langle \text{length} \rangle$,</code>	
<code>default : .3cm</code>	

continued ...

Table 24: ...continued

the radius of circular arcs used in curved edges of folded Dynkin diagrams.

`fold right` = $\langle \text{true or false} \rangle$,
 default : `true`
 whether to fold the roots on the right side of a Dynkin diagram.

`fold left style/.style` = $\langle \text{TikZ style data} \rangle$,
 default :
 style to override the `fold` style when folding roots together on the left half of a Dynkin diagram

`fold right style/.style` = $\langle \text{TikZ style data} \rangle$,
 default :
 style to override the `fold` style when folding roots together on the right half of a Dynkin diagram

`fold style/.style` = $\langle \text{TikZ style data} \rangle$,
 default : `solid,draw=black!40,fill=none,line width=radius`
 when drawing folded diagrams, style for the fold indicators.

`indefinite edge ratio` = $\langle \text{float} \rangle$,
 default : 1.6
 ratio of indefinite edge lengths to other edge lengths.

`indefinite edge/.style` = $\langle \text{TikZ style data} \rangle$,
 default : `solid,draw=black,fill=white,thin,densely dotted`
 style of the dotted or dashed middle third of each indefinite edge.

`Kac` = $\langle \text{true or false} \rangle$,
 default : `false`
 whether to draw in the style of [15]

`label` = `true or false`,
 default : `false`
 whether to label the roots according to the current labelling scheme.

`label macro` = $\langle \text{1-parameter T}_{\text{E}}\text{X macro} \rangle$,
 default : `#1`
 the current labelling scheme for roots.

`label macro*` = $\langle \text{1-parameter T}_{\text{E}}\text{X macro} \rangle$,
 default : `#1`
 the current labelling scheme for alternate roots.

`label height` = $\langle \text{1-parameter T}_{\text{E}}\text{X macro} \rangle$,
 default : `b`
 the current maximal height of text labels for the roots, set by giving mathematics text of that height.

`label depth` = $\langle \text{1-parameter T}_{\text{E}}\text{X macro} \rangle$,
 default : `g`
 the current maximal depth of text labels for the roots, set by giving mathematics text of that depth.

`make indefinite edge` = $\langle \text{edge pair } i\text{-}j \text{ or list of such} \rangle$,
 default : `{}`
 edge pair or list of edge pairs to treat as having indefinitely many roots on them.

continued ...

Table 24: ...continued

mark = $\langle o, O, t, x, X, * \rangle$,	
default : *	default root mark
name = $\langle \text{string} \rangle$,	
default : anonymous	A name for the Dynkin diagram, with anonymous treated as a blank; see section 26.
ordering = $\langle \text{Adams, Bourbaki, Carter, Dynkin, Kac} \rangle$,	
default : Bourbaki	which ordering of the roots to use in exceptional root systems as in section 17.
parabolic = $\langle \text{integer} \rangle$,	
default : 0	A parabolic subgroup with specified integer, where the integer is computed as $n = \sum 2^{i-1} a_i$, $a_i = 0$ or 1, to say that root i is crossed, i.e. a noncompact root.
ply = $\langle 0, 1, 2, 3, 4 \rangle$,	
default : 0	how many roots get folded together, at most.
reverse arrows = $\langle \text{true or false} \rangle$,	
default : true	whether to reverse the direction of the arrows that arise along the edges.
root radius = $\langle \text{number} \rangle \text{cm}$,	
default : .05cm	size of the dots and of the crosses in the Dynkin diagram
text style = $\langle \text{TikZ style data} \rangle$,	
default : scale=.7	Style for any labels on the roots.
upside down = $\langle \text{true or false} \rangle$,	
default : false	whether to reverse up to down.

All other options are passed to TikZ.

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SCHOOL OF MATHEMATICAL SCIENCES, UNIVERSITY COLLEGE CORK, CORK, IRELAND

Email address: b.mckay@ucc.ie