

**THE DYNKIN DIAGRAMS PACKAGE**  
**VERSION 3.141 592 653 589 793 23**

BEN MCKAY

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## 1. QUICK INTRODUCTION

Load the Dynkin diagram package (see options below)

```
\documentclass{amsart}
\usepackage{dynkin-diagrams}
\begin{document}
The Dynkin diagram of  $(B_3)$  is \dynkin B3.
\end{document}
```

Invoke it

The Dynkin diagram of  $(B_3)$  is \dynkin B3.

The Dynkin diagram of  $B_3$  is .


Indefinite rank Dynkin diagrams

```
\dynkin B{}
```



Inside a TikZ statement

```
The Dynkin diagram of  $(B_3)$  is
\tikz \dynkin B3;
```

The Dynkin diagram of  $B_3$  is .

Inside a Dynkin diagram environment

```
The Dynkin diagram of  $(B_3)$  is
\begin{dynkinDiagram}B3
\draw[very thick,red] (root 1) to [out=-45, in=-135] (root 3);
\end{dynkinDiagram}
```

The Dynkin diagram of  $B_3$  is .

## 2. INTERACTION WITH TIKZ

Inside a *TikZ* environment, default behaviour is to draw from the origin, so you can draw around the diagram:

Inside a *TikZ* environment

```
\begin{tikzpicture}
\draw (0,0) -- (.5,1) -- (1,0);
\dynkin[edge length=1cm]G2
\end{tikzpicture}
```



But it looks bad in the middle of text:

Inside a *TikZ* environment

```
The Dynkin diagram of \(\mathbf{B}_3\) is
\begin{tikzpicture}[baseline]
\dynkin B3
\draw[very thick,red] (root 1) to [out=-45, in=-135] (root 3);
\end{tikzpicture}
```

The Dynkin diagram of  $B_3$  is 




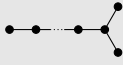





A vertical shift realigns the diagram to ambient text:

Inside a *TikZ* environment

```
The Dynkin diagram of \(\mathbf{B}_3\) is
\begin{tikzpicture}[baseline]
\dynkin[vertical shift] B3
\draw[very thick,red] (root 1) to [out=-45, in=-135] (root 3);
\end{tikzpicture}
```

The Dynkin diagram of  $B_3$  is 

Table 1: The Dynkin diagrams of the reduced simple root systems  
[3] pp. 265–290, plates I–IX

$A_n$		<code>\dynkin A{}</code>
$B_n$		<code>\dynkin B{}</code>
$C_n$		<code>\dynkin C{}</code>
$D_n$		<code>\dynkin D{}</code>
$E_6$		<code>\dynkin E6</code>
$E_7$		<code>\dynkin E7</code>
$E_8$		<code>\dynkin E8</code>
$F_4$		<code>\dynkin F4</code>
$G_2$		<code>\dynkin G2</code>

### 3. SET OPTIONS GLOBALLY

Most options set globally ...

```
\pgfkeys{/Dynkin diagram,
  edge length=.5cm,
  fold radius=.5cm,
  indefinite edge/.style={
    draw=black,
    fill=white,
    thin,
    densely dashed}}
```

You can also pass options to the package in `\usepackage`. *Danger*: spaces in option names are replaced with hyphens: `edge length=1cm` is `edge-length=1cm` as a global option; moreover you should drop the extension `/.style` on any option with spaces in its name (but not otherwise). For example,

...or pass global options to the package


```
\usepackage[
  ordering=Kac,
  edge/.style=blue,
  indefinite-edge={draw=green,fill=white,densely dashed},
  indefinite-edge-ratio=5,
  mark=o,
  root-radius=.06cm]
{dynkin-diagrams}
```

## 4. DISCONNECTED DYNKIN DIAGRAMS

Disconnected Dynkin diagrams that represent a product of simple Lie groups (or a sum of Lie algebras, or a product of Coxeter systems, ...) have a different syntax (to ensure back compatibility):

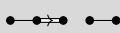
## Command

The Dynkin diagram of  $(B_3 \times A_2)$  is `\dynkins{B3|A2}`.

The Dynkin diagram of  $B_3 \times A_2$  is .

## Environment

The Dynkin diagram of  $(B_3 \times A_2)$  is  
`\begin{DynkinDiagrams}{B3|A2}\end{DynkinDiagrams}`

The Dynkin diagram of  $B_3 \times A_2$  is .

Each factor can have its own options.

## Environment





The Dynkin diagram of  $(B_3 \times A_2)$  is  
`\[`  
`\begin{DynkinDiagrams}{[name=Bob]B3|[name=Alice]A2}`  
`\draw[very thick,blue] (Bob root 1)`  
`to [out=-45, in=-135] (Alice root 2);`  
`\end{DynkinDiagrams}`  
`\]`

The Dynkin diagram of  $B_3 \times A_2$  is





They are spaced out by the length of one edge between successive diagrams; change this with `separator length`.

Table 2: The Dynkin diagrams of the rank 2 root systems

$A_1 \times A_1$		<code>\dynkins {A1 A1}</code>
$A_2$		<code>\dynkins {A2}</code>
$B_2$		<code>\dynkins {B2}</code>
$C_2$		<code>\dynkins {C2}</code>

continued ...

Table 2: ...continued

$D_2$		<code>\dynkins {D2}</code>
$G_2$		<code>\dynkins {G2}</code>

## 5. COXETER DIAGRAMS


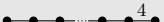
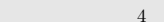

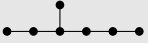
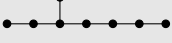





Coxeter diagram option

`\dynkin[Coxeter]F4`gonality option for  $G_2$  and  $I_n$  Coxeter diagrams
$$\backslash(G_2=\backslash\text{dynkin}[Coxeter,gonality=n]G2\backslash), \backslash$$

$$\backslash(I_n=\backslash\text{dynkin}[Coxeter,gonality=n]I\backslash\backslash)$$

$$G_2 = \overset{n}{\bullet} \bullet, \quad I_n = \overset{n}{\bullet} \bullet$$

Table 3: The Coxeter diagrams of the simple reflection groups

$A_n$		<code>\dynkin [Coxeter]A{}</code>
$B_n$		<code>\dynkin [Coxeter]B{}</code>
$C_n$		<code>\dynkin [Coxeter]C{}</code>
$E_6$		<code>\dynkin [Coxeter]E6</code>
$E_7$		<code>\dynkin [Coxeter]E7</code>
$E_8$		<code>\dynkin [Coxeter]E8</code>
$F_4$		<code>\dynkin [Coxeter]F4</code>
$G_2$		<code>\dynkin [Coxeter,gonality=n]G2</code>
$H_3$		<code>\dynkin [Coxeter]H3</code>
$H_4$		<code>\dynkin [Coxeter]H4</code>
$I_n$		<code>\dynkin [Coxeter,gonality=n]I{}</code>

## 6. SATAKE DIAGRAMS

Satake diagrams use the standard name instead of a rank

`\(A_{IIIb}=\dynkin A{IIIb}\)`

$$A_{IIIb} = \begin{array}{c} \circ - \circ - \circ - \circ \\ | \quad | \quad | \quad | \\ \circ - \circ - \circ - \circ \end{array}$$


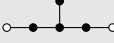
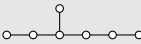


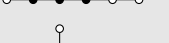
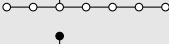



We use a solid gray bar to denote the folding of a Dynkin diagram, rather than the usual double arrow, since the diagrams turn out simpler and easier to read.

Table 4: The Satake diagrams of the real simple Lie algebras [13] p. 532–534

$A_I$		<code>\dynkin AI</code>
$A_{II}$		<code>\dynkin A{II}</code>
$A_{IIIa}$		<code>\dynkin A{IIIa}</code>
$A_{IIIb}$		<code>\dynkin A{IIIb}</code>
$A_{IV}$		<code>\dynkin A{IV}</code>
$B_I$		<code>\dynkin BI</code>
$B_{II}$		<code>\dynkin B{II}</code>
$C_I$		<code>\dynkin CI</code>
$C_{IIa}$		<code>\dynkin C{IIa}</code>
$C_{IIb}$		<code>\dynkin C{IIb}</code>
$D_{Ia}$		<code>\dynkin D{Ia}</code>
$D_{Ib}$		<code>\dynkin D{Ib}</code>
$D_{Ic}$		<code>\dynkin D{Ic}</code>
$D_{II}$		<code>\dynkin D{II}</code>
$D_{IIIa}$		<code>\dynkin D{IIIa}</code>
$D_{IIIb}$		<code>\dynkin D{IIIb}</code>
$E_I$		<code>\dynkin EI</code>
$E_{II}$		<code>\dynkin E{II}</code>

continued ...

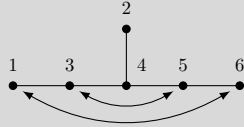
Table 4: ...continued

$E_{III}$		<code>\dynkin E{III}</code>
$E_{IV}$		<code>\dynkin E{IV}</code>
$E_V$		<code>\dynkin EV</code>
$E_{VI}$		<code>\dynkin E{VI}</code>
$E_{VII}$		<code>\dynkin E{VII}</code>
$E_{VIII}$		<code>\dynkin E{VIII}</code>
$E_{IX}$		<code>\dynkin E{IX}</code>
$F_I$		<code>\dynkin FI</code>
$F_{II}$		<code>\dynkin F{II}</code>
$G_I$		<code>\dynkin GI</code>

## 7. HOW TO FOLD

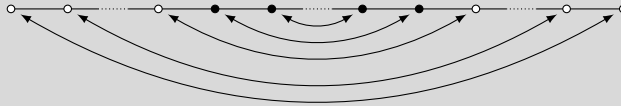
If you don't like the solid gray "folding bar", most people use arrows. Here is  $E_{II}$

```
\dynkin[edge length=.75cm,
labels*={1,...,6},
involutions={16;35}]E6
```



The double arrows for  $A_{IIIa}$  are big

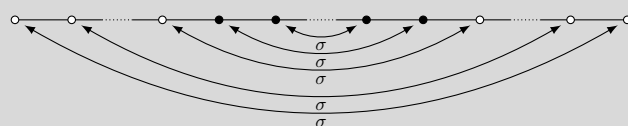
```
\dynkin[edge length=.75cm,
involutions={1{10};29;38;47;56}]{A}{oo.o**.**o.oo}
```





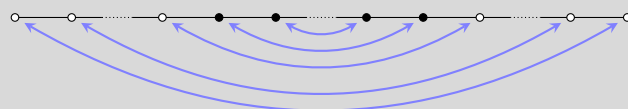
We can add labels

```
\dynkin[edge length=.75cm,
  involutions={
    1<below>[\sigma]{10};
    2<below>[\sigma]9;
    3<below>[\sigma]8;
    4<below>[\sigma]7;
    5<below>[\sigma]6}
]{A}{oo.o**.**o.oo}
```



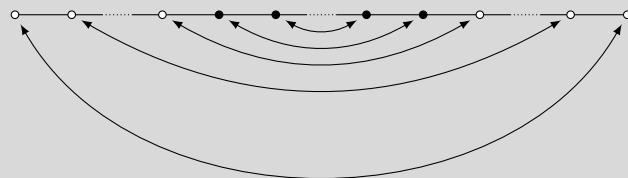
Style options

```
\dynkin[edge length=.75cm,
  involution/.style={blue!50,stealth-stealth,thick},
  involutions={1{10};29;38;47;56}
]{A}{oo.o**.**o.oo}
```



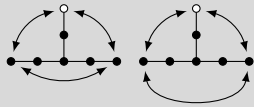
Arrow angles

```
\dynkin[edge length=.75cm,
  involutions={[\in=-120,out=-60,relative]1{10};29;38;47;56}
]{A}{oo.o**.**o.oo}
```



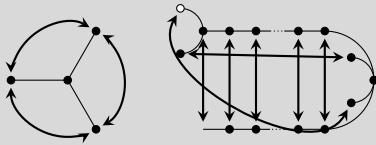
## Arrow angles

```
\dynkin[involutions={16;60;01}]E[1]{6}
\dynkin[involutions={[out=-80,in=-100,relative]16;60;01}]E[1]{6}
```



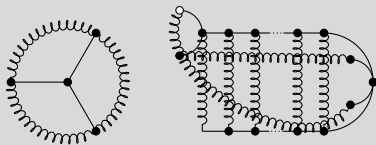
If you don't like the solid gray "folding bar", most people use arrows ...

```
\tikzset{/Dynkin diagram/fold style/.style={stealth-stealth,thick,
shorten <=1mm,shorten >=1mm,}}
\dynkin[ply=3,edge length=.75cm]D4
\begin{dynkinDiagram}[ply=4]D[1]%
{****.*****.*****}
\dynkinFold 1{13}
\dynkinFold[bend right=90] 0{14}
\end{dynkinDiagram}
```



...but you could try springs pulling roots together

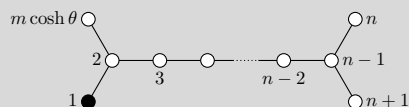
```
\tikzset{/Dynkin diagram/fold style/.style=
{decorate,decoration={name=coil,aspect=0.5,
segment length=1mm,amplitude=.6mm}}}
\dynkin[ply=3,edge length=.75cm]D4
\begin{dynkinDiagram}[ply=4]D[1]%
{****.*****.*****}
\dynkinFold 1{13}
\dynkinFold[bend right=90]0{14}
\end{dynkinDiagram}
```



## 8. LABELS FOR THE ROOTS

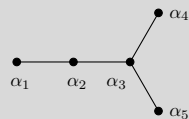
Make a list of labels for the roots. Optionally, you can add label directions to say where to put each label relative to its root.

```
\dynkin[labels={m\cosh\theta,1,2,3,,n-2,n-1,n,n+1},
          label directions={,,left,,,right,,},
          scale=1.8,
          extended] D{*ooo...oooo}
```



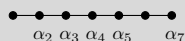
Make a macro to assign labels to roots

```
\dynkin[label,label macro/.code={\alpha_{\drlap{#1}}},edge
          length=.75cm]D5
```



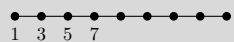
Labelling several roots

```
\dynkin[labels={,2,...,5,,7},label
          macro/.code={\alpha_{\drlap{#1}}}]A7
```



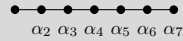
The foreach notation I

```
\dynkin[labels={1,3,...,7}]A9
```



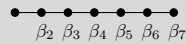
## The foreach notation II

```
\dynkin[labels={,\alpha_2,\alpha_...\alpha_7}]A7
```



## The foreach notation III

```
\dynkin[label macro/.code={\beta_{\drlap{#1}}},labels={,2,...,7}]A7
```



## Label the roots individually by root number

```
\dynkin[label]B3
```



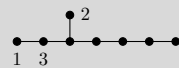
## Access root labels via TikZ

```
\begin{dynkinDiagram}B3
\node[below,/Dynkin diagram/text style] at (root 2)
{\(\alpha_{\drlap{2}}\)};
\end{dynkinDiagram}
```



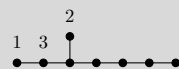
## The labels have default locations, mostly below roots

```
\dynkin[labels={1,2,3}]E8
```



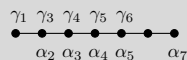
## The starred form flips labels to alternate locations, mostly above roots

```
\dynkin[labels*={1,2,3}]E8
```



## Labelling several roots and alternates

```
\dynkin[label macro/.code={\alpha_{\drlap{#1}}},
        label macro*/.code={\gamma_{\drlap{#1}}},
        labels={,2,...,5,,7},
        labels*={1,3,4,5,6}]A7
```

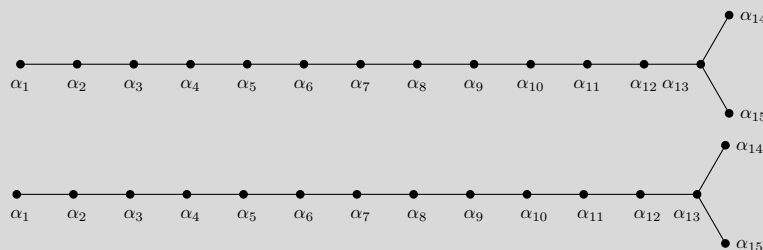


## 9. LABEL SUBSCRIPTS

Note the slight improvement that `\drlap` makes: the labels are centered on the middle of the letter  $\alpha$ , ignoring the space taken up by the subscripts, using the `mathtools` command `\mathrlap`, but only for labels which are *not* placed to the left or right of a root.

## Label subscript spacing

```
\dynkin[label,label macro/.code={\alpha_{#1}},
        edge length=.75cm]D{15}
\par\noindent{}%
\dynkin[label,label macro/.code={\alpha_{\drlap{#1}}},
        edge length=.75cm]D{15}
```

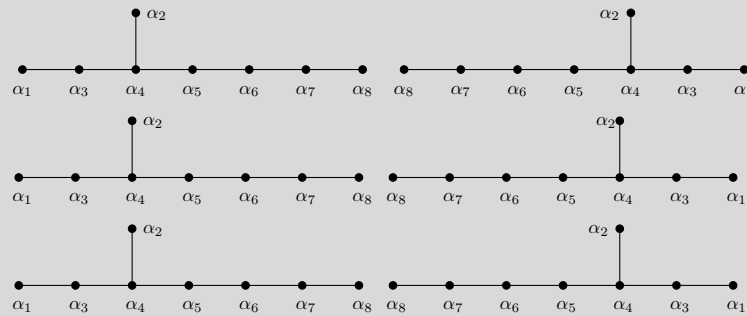


## Label subscript spacing

```

\dynkin[label,label macro/.code={\alpha_{#1}},
        edge length=.75cm]E8
\dynkin[label,label macro/.code={\alpha_{#1}},backwards,
        edge length=.75cm]E8
\par\noindent{}%
\dynkin[label,label macro/.code={\alpha_{\mathrlap{#1}}},
        edge length=.75cm]E8
\dynkin[label,label macro/.code={\alpha_{\mathrlap{#1}}},backwards,
        edge length=.75cm]E8
\par\noindent{}%
\dynkin[label,label macro/.code={\alpha_{\drlap{#1}}},
        edge length=.75cm]E8
\dynkin[label,label macro/.code={\alpha_{\drlap{#1}}},backwards,
        edge length=.75cm]E8

```

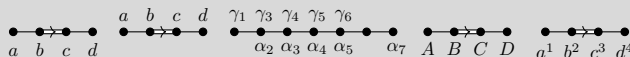


## 10. HEIGHT AND DEPTH OF LABELS

Labels are set with default maximum height the height of the character  $b$ , and default maximum depth the depth of the character  $g$ . To change these, set `label height` and `label depth`:

Change height and depth of characters

```
\dynkin[labels={a,b,c,d},label height=d,label depth=d]F4
\dynkin[labels*={a,b,c,d},label height=d,label depth=d]F4
\dynkin[label macro/.code={\alpha_{\drlap{#1}}},
        label macro*/.code={\gamma_{\drlap{#1}}},
        label height=$\alpha_1$,
        label depth=$\alpha_1$,
        labels={,2,...,5,,7},
        labels*={1,3,4,5,6}]A7
\dynkin[labels={A,B,C,D},label height=$A$,label depth=$A$]F4
\dynkin[labels={a^1,b^2,c^3,d^4},label height=$X^X$]F4
```



## 11. TEXT STYLE FOR THE LABELS

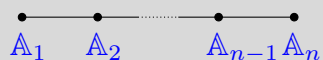
Use a text style: big and blue

```
\begin{dynkinDiagram}[text style/.style={scale=1.2,blue},
    edge length=1cm,
    labels={1,2,n-1,n},
    label macro/.code={\alpha_{\drlap{#1}}}]A{}
\end{dynkinDiagram}
```



Use a text style; font selection is in the label macro

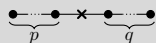
```
\begin{dynkinDiagram}[text style/.style={scale=1.2,blue},
  edge length=1cm,
  labels={1,2,n-1,n},
  label macro/.code={\mathbb{A}_{\drlap{#1}}}]A{}
\end{dynkinDiagram}
```



## 12. BRACING ROOTS

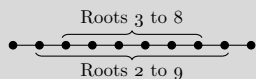
Bracing roots

```
\begin{dynkinDiagram}A{*.x*.x*}
  \dynkinBrace[p]12
  \dynkinBrace[q]45
\end{dynkinDiagram}
```



Bracing roots, and a starred form

```
\begin{dynkinDiagram}A{10}
  \dynkinBrace[\text{Roots 2 to 9}]29
  \dynkinBrace*[\text{Roots 3 to 8}]38
\end{dynkinDiagram}
```



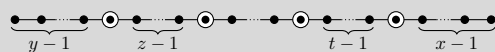


## Bracing roots

```

\newcommand\circleRoot[1]{
\draw[fill=white] (root #1) circle (3pt);
\fill[black] (root #1) circle (1.5pt);}
\begin{dynkinDiagram}A{**.***.***.***.***.**}
\foreach\r in {4,7,10,13} {\circleRoot \r}
\dynkinBrace[y-1]13
\dynkinBrace[z-1]56
\dynkinBrace[t-1]{11}{12}
\dynkinBrace[x-1]{14}{16}
\end{dynkinDiagram}

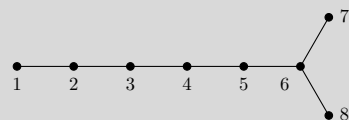
```



## 13. LABEL PLACEMENT

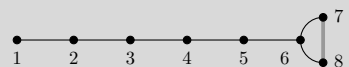
Take a  $D_8$ :

```
\dynkin[label,edge length=.75cm]D8
```



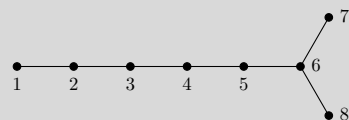
If you want to fold this diagram,

```
\dynkin[fold right,label,edge length=.75cm]D8
```



you will be glad that the 6 sits where it does, under and to the left. If you don't want to fold, you might prefer instead to put the 6 on the right side.

```
\dynkin[label,edge length=.75cm,label directions={,,,right,,}]D8
```



The default locations are overridden by the `label directions`. For extended diagrams, this list starts at 0-offset.

```
\dynkin[label,
  label directions={above,,,,,},
  involutions={ [out=-60,in=-120,relative] 16;60;01}
]E[1]{6}
```

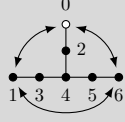
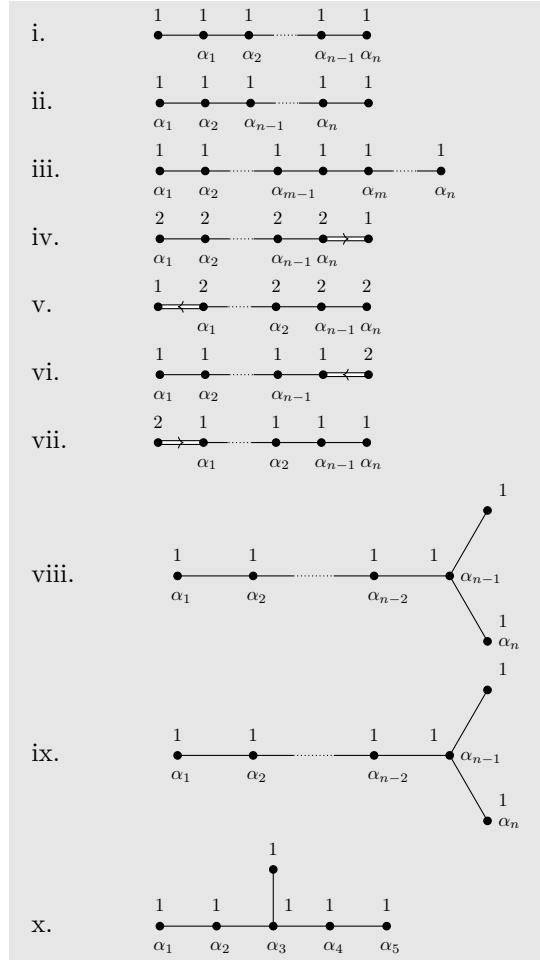
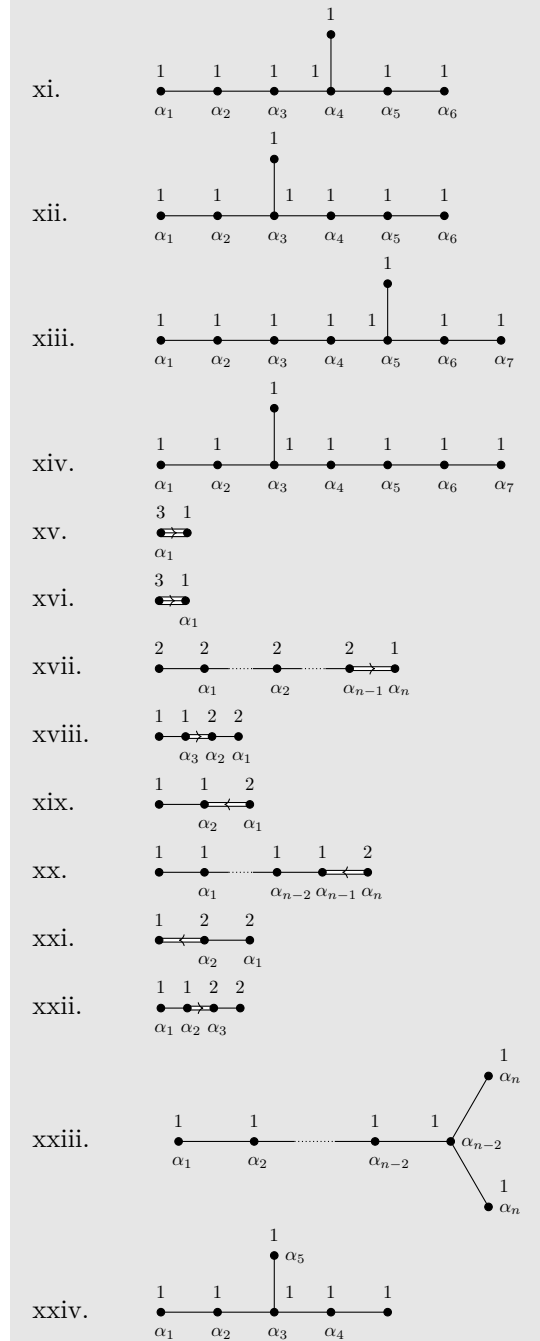


Table 5: Dynkin diagrams from Euler products [18]



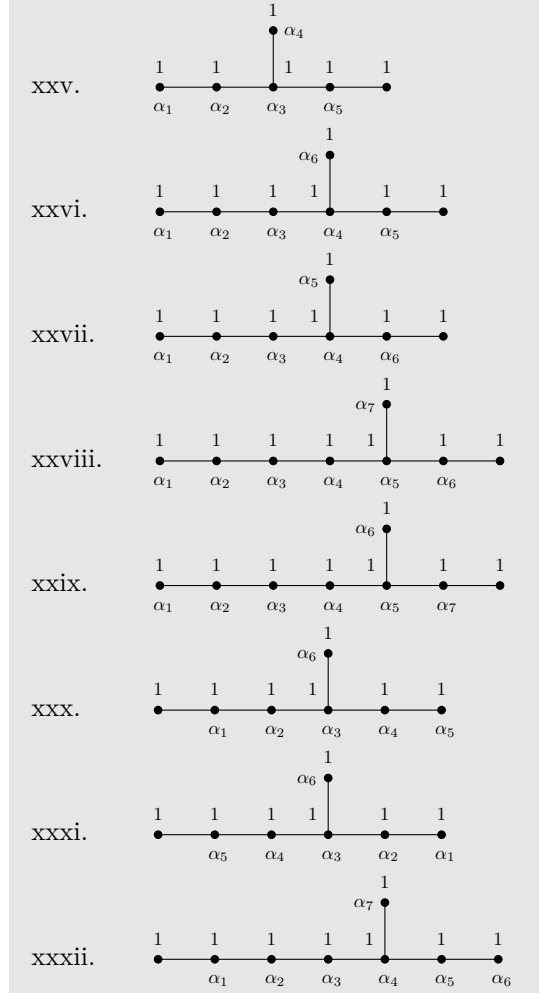
continued ...

Table 5: ...continued



continued ...

Table 5: ...continued



```

\tikzset{/Dynkin diagram,ordering=Dynkin,label macro/.code={\alpha_{\drlap{#1}}}}
\newcounter{EPNo}
\setcounter{EPNo}{0}
\NewDocumentCommand\EP{smmmm}{
  \stepcounter{EPNo}\roman{EPNo}. &
  \def\eL{.6cm}
  \IfStrEqCase{#2}{
    D{
      \gdef\eL{1cm}
      \tikzset{/Dynkin diagram/label directions={,,right,,}}
      E{\gdef\eL{.75cm}}
      F{\gdef\eL{.35cm}}
      G{\gdef\eL{.35cm}}
    }
    \IfBooleanTF{#1}{
      \dynkin[edge length=\eL,backwards,labels*={#4},labels={#5}]{#2}{#3}
    }{
  }
}

```

```

\dynkin[edge length=\eL,labels*={#4},labels={#5}]{#2}{#3}
\tikzset{/Dynkin diagram/label directions={}}
\}
\renewcommand*\do[1]{\EP#1}%
\begin{longtable}{MM}
\caption{Dynkin diagrams from Euler products \cite{Langlands:1967}}\
\endfirsthead
\caption{\dots continued}\
\endhead
\multicolumn{2}{c}{continued \dots}\
\endfoot
\endlastfoot
\docsvlist{
A{***.***}{1,1,1,1,1}{1,2,n-1,n},
A{***.***}{1,1,1,1,1}{1,2,n-1,n},
A{**.*.***}{1,1,1,1,1,1}{1,2,m-1,,m,n},
B{**.*.***}{2,2,2,2,1}{1,2,n-1,n},
*B{***.***}{2,2,2,2,1}{n,n-1,2,1,},
C{**.*.***}{1,1,1,1,2}{1,2,n-1,},
*C{***.***}{1,1,1,1,2}{n,n-1,2,1,},
D{**.*.***}{1,1,1,1,1,1}{1,2,n-2,n-1,n},
D{**.*.***}{1,1,1,1,1,1}{1,2,n-2,n-1,n},
E6{1,1,1,1,1,1}{1,...,5},
*E7{1,1,1,1,1,1,1}{6,...,1},
E7{1,1,1,1,1,1,1}{1,...,6},
*E8{1,1,1,1,1,1,1,1}{7,...,1},
E8{1,1,1,1,1,1,1,1}{1,...,7},
G2{1,3}{1,},
G2{1,3}{1,},
B{**.*.***}{2,2,2,2,1}{1,2,n-1,n},
F4{1,1,2,2}{3,2,1},
C3{1,1,2}{2,1},
C{**.*.***}{1,1,1,1,2}{1,n-2,n-1,n},
*B3{2,2,1}{1,2},
F4{1,1,2,2}{1,2,3},
D{**.*.***}{1,1,1,1,1,1}{1,2,n-2,n-2,n,n},
E6{1,1,1,1,1,1}{1,2,3,4,,5},
E6{1,1,1,1,1,1}{1,2,3,5,,4},
*E7{1,1,1,1,1,1,1}{5,...,1,6},
*E7{1,1,1,1,1,1,1,1}{6,4,3,2,1,5},
*E8{1,1,1,1,1,1,1,1}{6,...,1,7},
*E8{1,1,1,1,1,1,1,1}{7,5,4,3,2,1,6},
*E7{1,1,1,1,1,1,1,1}{5,...,1,,6},
*E7{1,1,1,1,1,1,1,1}{1,...,5,,6},
*E8{1,1,1,1,1,1,1,1}{6,...,1,,7}}
\end{longtable}

```

## 14. STYLE

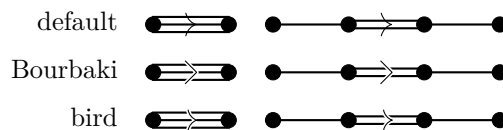
## Colours

```
\dynkin[extended,
  o/.append style={fill=orange},
  */.style=blue!50!red,
  edge length=.75cm,
  edge/.style={blue!50,thick},
  arrow width=2mm,
  arrow style={red,width=2mm,line width=1pt}]F4
```



Popular arrow shapes. These mess with nonwhite backgrounds, but are prettier than the default shape.

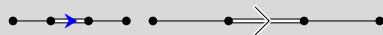
```
\begin{tcolorbox}[colback=white,colframe=white]
\begin{tabular}{rcc}
default&\dynkin G2 & \dynkin F4\\
Bourbaki&\dynkin[Bourbaki arrow]G2&\dynkin[Bourbaki arrow]F4\\
bird&\dynkin[bird arrow]G2 & \dynkin[bird arrow]F4
\end{tabular}
\end{tcolorbox}
```



Use `\tikzset{/Dynkin diagram,Bourbaki arrow}` to force all arrows to have Bourbaki style throughout your document.


## Other arrow shapes

```
\dynkin[edge length=.5cm,
  arrow width=2mm,
  arrow shape/.style={-Stealth[blue,width=2mm]}}F4
\dynkin[edge length=1cm,
  arrow shape/.style={-Bourbaki[length=7pt]}}F4
```



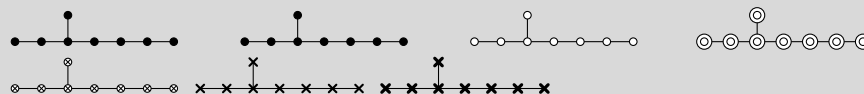
## Edge lengths

The Dynkin diagram of  $\backslash(A_3\backslash)$  is `\dynkin[edge length=1.2]A3`

The Dynkin diagram of  $A_3$  is 

## Root marks

`\dynkin E8`  
`\dynkin[mark=*] E8`  
`\dynkin[mark=o] E8`  
`\dynkin[mark=O] E8`  
`\dynkin[mark=t] E8`  
`\dynkin[mark=x] E8`  
`\dynkin[mark=X] E8`




At the moment, you can only use:

- \* • solid dot
- o ○ hollow circle
- O ⊙ double hollow circle
- t ⊗ tensor root
- x × crossed root
- X × thickly crossed root

## Mark styles

The parabolic subgroup  $\backslash(E_{8,124}\backslash)$  is  
`\dynkin[parabolic=124,x/.style={brown,very thick}]E8`

The parabolic subgroup  $E_{8,124}$  is 

## Sizes of root marks

$\backslash(A_{3,3}\backslash)$  with big root marks is `\dynkin[root radius=.08cm,parabolic=3]A3`

$A_{3,3}$  with big root marks is 

## 15. SUPPRESS OR REVERSE ARROWS

Some diagrams have double or triple edges

```
\dynkin F4
\dynkin G2
```



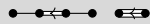
Suppress arrows

```
\dynkin[arrows=false]F4
\dynkin[arrows=false]G2
```



Reverse arrows

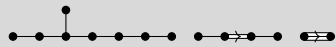
```
\dynkin[reverse arrows]F4
\dynkin[reverse arrows]G2
```



## 16. BACKWARDS AND UPSIDE DOWN

Default

```
\dynkin E8
\dynkin F4
\dynkin G2
```



Backwards

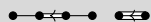
```
\dynkin[backwards]E8
\dynkin[backwards]F4
\dynkin[backwards]G2
```





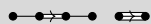
## Reverse arrows

```
\dynkin[reverse arrows]F4
\dynkin[reverse arrows]G2
```



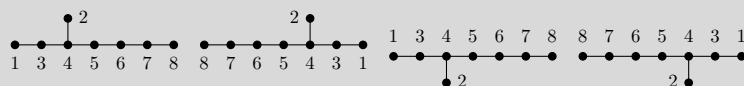
## Backwards, reverse arrows

```
\dynkin[backwards,reverse arrows]F4
\dynkin[backwards,reverse arrows]G2
```



## Backwards versus upside down

```
\dynkin[label]E8
\dynkin[label,backwards]E8
\dynkin[label,upside down]E8
\dynkin[label,backwards,upside down]E8
```



## 17. DRAWING ON TOP OF A DYNKIN DIAGRAM

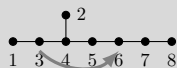
## TikZ can access the roots themselves

```
\begin{dynkinDiagram}A4
  \fill[white,draw=black] (root 2) circle (.15cm);
  \fill[white,draw=black] (root 2) circle (.1cm);
  \draw[black] (root 2) circle (.05cm);
\end{dynkinDiagram}
```



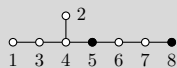
Draw curves between the roots

```
\begin{dynkinDiagram}[label]E8
  \draw[very thick, black!50,-latex]
    (root 3.south) to [out=-45, in=-135] (root 6.south);
\end{dynkinDiagram}
```



Change marks

```
\begin{dynkinDiagram}[mark=o,label]E8
  \dynkinRootMark{*}5
  \dynkinRootMark{*}8
\end{dynkinDiagram}
```



## 18. MARK LISTS

The package allows a list of root marks instead of a rank:

A mark list

```
\dynkin E{oo**ttxx}
```



The mark list `oo**ttxx` has one mark for each root: `o`, `o`,  $\dots$ , `x`. Roots are listed in the current default ordering. (Careful: in an affine root system, a mark list will *not* contain a mark for root zero.)

If you need to repeat a mark, you can give a *single digit* positive integer to indicate how many times to repeat it.

A mark list with repetitions

```
\dynkin A{x4o3t4}
```

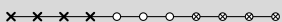


Table 6: Classical Lie superalgebras [10]. We need a slightly larger root radius parameter to distinguish the tensor product symbols from the solid dots.

		<code>\tikzset{/Dynkin diagram,root radius=.07cm}</code>
$A_{mn}$		<code>\dynkin A{o3.oto.oo}</code>
$B_{mn}$		<code>\dynkin B{o3.oto.oo}</code>
$B_{0n}$		<code>\dynkin B{o3.o3.o*}</code>
$C_n$		<code>\dynkin C{too.oto.oo}</code>
$D_{mn}$		<code>\dynkin D{o3.oto.o4}</code>
$D_{21\alpha}$		<code>\dynkin A{oto}</code>
$F_4$		<code>\dynkin F{ooot}</code>
$G_3$		<code>\dynkin [extended,affine mark=t, reverse arrows]G2</code>

Table 7: Classical Lie superalgebras [10]. Here we see the problem with using the default root radius parameter, which is too small for tensor product symbols.

$A_{mn}$		<code>\dynkin A{o3.oto.oo}</code>
$B_{mn}$		<code>\dynkin B{o3.oto.oo}</code>
$B_{0n}$		<code>\dynkin B{o3.o3.o*}</code>
$C_n$		<code>\dynkin C{too.oto.oo}</code>
$D_{mn}$		<code>\dynkin D{o3.oto.o4}</code>
$D_{21\alpha}$		<code>\dynkin A{oto}</code>
$F_4$		<code>\dynkin F{ooot}</code>
$G_3$		<code>\dynkin [extended,affine mark=t, reverse arrows]G2</code>

## 19. INDEFINITE EDGES

An *indefinite edge* is a dashed edge between two roots,  $\bullet \cdots \bullet$  indicating that an indefinite number of roots have been omitted from the Dynkin diagram. In between any two entries in a mark list, place a period to indicate an indefinite edge:

Indefinite edges

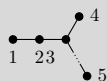
`\dynkin D{o.o*.*.t.to.t}`



In certain diagrams, roots may have an edge between them even though they are not subsequent in the ordering. For such rare situations, there is an option:

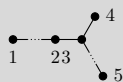
#### Indefinite edge option

```
\dynkin[make indefinite edge={3-5},label]D5
```



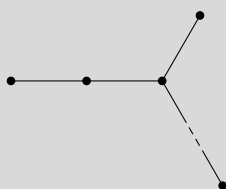
#### Give a list of edges to become indefinite

```
\dynkin[make indefinite edge/.list={1-2,3-5},label]D5
```



#### Indefinite edge style

```
\dynkin[indefinite edge/.style={
    draw=black,fill=white,thin,densely dashed},
    edge length=1cm,
    make indefinite edge={3-5}]D5
```



#### The ratio of the lengths of indefinite edges to those of other edges

```
\dynkin[edge length = .5cm,
    indefinite edge ratio=3,
    make indefinite edge={3-5}]D5
```

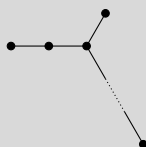


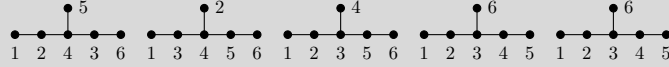
Table 8: Springer's table of indices [25], pp. 320-321, with one form of  $E_7$  corrected

$A_n$		
$A_n$		
$B_n$		
$C_n$		
$D_n$		
$E_6$		<code>\dynkin E{*oooo*}</code>
$E_6$		<code>\dynkin E{o*o*oo}</code>
$E_6$		<code>\dynkin E{o*oooo}</code>
$E_6$		<code>\dynkin E{**ooo*}</code>
$E_7$		<code>\dynkin E{*oooooo}</code>
$E_7$		<code>\dynkin E{oooooo*o}</code>
$E_7$		<code>\dynkin E{oooooo*}</code>
$E_7$		<code>\dynkin E{*oooo*o}</code>
$E_7$		<code>\dynkin E{*oooo**}</code>
$E_7$		<code>\dynkin E{*o**o*o}</code>
$E_8$		<code>\dynkin E{*oooooooo}</code>
$E_8$		<code>\dynkin E{oooooooo*}</code>
$E_8$		<code>\dynkin E{*oooooooo*}</code>
$E_8$		<code>\dynkin E{oooooooo**}</code>
$E_8$		<code>\dynkin E{*oooo***}</code>
$F_4$		<code>\dynkin F{ooo*}</code>
$D_4$		<code>\dynkin D{o*oo}</code>

## 20. ROOT ORDERING

Root ordering

```
\dynkin[label,ordering=Adams]E6
\dynkin[label,ordering=Bourbaki]E6
\dynkin[label,ordering=Carter]E6
\dynkin[label,ordering=Dynkin]E6
\dynkin[label,ordering=Kac]E6
```

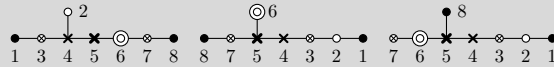


Default is Bourbaki. Sources are Adams [1] p. 56–57, Bourbaki [3] p. pp. 265–290 plates I–IX, Carter [5] p. 540–609, Dynkin [8], Kac [16] p. 43.

	Adams	Bourbaki	Carter	Dynkin	Kac
$E_6$					
$E_7$					
$E_8$					
$F_4$					
$G_2$					

The marks are set down in order according to the current root ordering:

```
\dynkin[label]E{*otxX0t*}
\dynkin[label,ordering=Carter]E{*otxX0t*}
\dynkin[label,ordering=Kac]E{*otxX0t*}
```



## Convert between orderings

```
\newcount\r
\dynkinOrder E8.Carter::6->Bourbaki.{\r}
In \(\mathbb{E}_8\), root 6 in Carter's ordering is root \(\mathbb{r}\) in
    Bourbaki's ordering.
```

In  $E_8$ , root 6 in Carter's ordering is root 2 in Bourbaki's ordering.

## 21. PARABOLIC SUBGROUPS

Each set of roots is assigned a number, with each binary digit zero or one to say whether the corresponding root is crossed or not:

The flag variety of pointed lines in projective 3-space is associated to the Dynkin diagram `\dynkin[parabolic=3]A3`.

The flag variety of pointed lines in projective 3-space is associated to the Dynkin diagram  $\times \leftarrow \times \rightarrow \bullet$ .

## Commutative diagrams: anchor nodes to center

```
\begin{tikzcd}[row sep=0em,column sep=1em,cramped,
cells={nodes={anchor=center}}]
& \dynkin{G}{xx} \arrow{dr} \arrow{dl} & \\
& \dynkin{G}{*x} \arrow{dr} & \\
& \dynkin{G}{x*} \arrow{dl} & \\
& \dynkin{G}{**} & \\
\end{tikzcd}
```

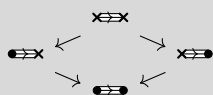


Table 10: The Hermitian symmetric spaces

$A_n$		Grassmannian of $k$ -planes in $\mathbb{C}^{n+1}$
$B_n$		$(2n - 1)$ -dimensional quadric hypersurface
$C_n$		space of Lagrangian $n$ -planes in $\mathbb{C}^{2n}$
$D_n$		$(2n - 2)$ -dimensional quadric hypersurface
$D_n$		component of maximal null subspaces of $\mathbb{C}^{2n}$
$D_n$		the other component
$E_6$		complexified octave projective plane
$E_6$		its dual plane
$E_7$		the space of null octave 3-planes in octave 6-space

```

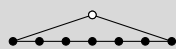
\NewDocumentCommand\HSS{mommm}
{#1&\IfNoValueTF{#2}{\dynkin{#3}{#4}}{\dynkin[parabolic=#2]{#3}{#4}}&#5\\}
\RenewDocumentCommand\do{m}{\HSS #1}
\renewcommand*{\arraystretch}{1.5}
\begin{longtable}
{>\columncolor[gray]{.9}}>$1<$>\columncolor[gray]{.9}}>$1<$>\columncolor[gray]{.9}}1}
\caption{The Hermitian symmetric spaces}\endhead\endfoot\endlastfoot
\docsvlist{%
{{A_n}A{**.*x**}}{Grassmannian of $k$-planes in $\mathbb{C}^{n+1}$}},
{{B_n}[1]B}{$(2n-1)$-dimensional quadric hypersurface}},
{{C_n}[16]C}{space of Lagrangian $n$-planes in $\mathbb{C}^{2n}$}},
{{D_n}[1]D}{$(2n-2)$-dimensional quadric hypersurface}},
{{D_n}[32]D}{component of maximal null subspaces of $\mathbb{C}^{2n}$}},
{{D_n}[16]D}{the other component}},
{{E_6}[1]E6}{complexified octave projective plane}},
{{E_6}[32]E6}{its dual plane}},
{{E_7}[64]E7}{the space of null octave 3-planes in octave 6-space}}}
\end{longtable}

```



## 22. EXTENDED DYNKIN DIAGRAMS

## Extended Dynkin diagrams

`\dynkin[extended]A7`

The extended Dynkin diagrams are also described in the notation of Kac [16] p. 55 as affine untwisted Dynkin diagrams: we extend `\dynkin A7` to become `\dynkin A[1]7`:

## Extended Dynkin diagrams

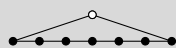
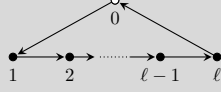
`\dynkin A[1]7`

Table 11: The Dynkin diagrams of the extended simple root systems

$A_1^1$		<code>\dynkin [extended]A1</code>
$A_n^1$		<code>\dynkin [extended]A{}</code>
$B_n^1$		<code>\dynkin [extended]B{}</code>
$C_n^1$		<code>\dynkin [extended]C{}</code>
$D_n^1$		<code>\dynkin [extended]D{}</code>
$E_6^1$		<code>\dynkin [extended]E6</code>
$E_7^1$		<code>\dynkin [extended]E7</code>
$E_8^1$		<code>\dynkin [extended]E8</code>
$F_4^1$		<code>\dynkin [extended]F4</code>
$G_2^1$		<code>\dynkin [extended]G2</code>

## Directed edges

```
\dynkin[edge length=.75cm,
  edge/.style={- stealth[sep=2pt]}},
  labels={,1,2,\ell-1,\ell},
  labels*={0}]A[1]{}
```



## 23. AFFINE TWISTED AND UNTWISTED DYNKIN DIAGRAMS

The affine Dynkin diagrams are described in the notation of Kac [16] p. 55:

## Affine Dynkin diagrams

```
\(A^{(1)}_7=\dynkin A[1]7, \
E^{(2)}_6=\dynkin E[2]6, \
D^{(3)}_4=\dynkin D[3]4\)
```

$$A_7^{(1)} = \text{diagram}, E_6^{(2)} = \text{diagram}, D_4^{(3)} = \text{diagram}$$

Table 12: The affine Dynkin diagrams

$A_1^1$		<code>\dynkin A[1]1</code>
$A_n^1$		<code>\dynkin A[1]{}</code>
$B_n^1$		<code>\dynkin B[1]{}</code>
$C_n^1$		<code>\dynkin C[1]{}</code>
$D_n^1$		<code>\dynkin D[1]{}</code>
$E_6^1$		<code>\dynkin E[1]6</code>
$E_7^1$		<code>\dynkin E[1]7</code>
$E_8^1$		<code>\dynkin E[1]8</code>
$F_4^1$		<code>\dynkin F[1]4</code>
$G_2^1$		<code>\dynkin G[1]2</code>
$A_2^2$		<code>\dynkin A[2]2</code>

continued ...

Table 12: ...continued

$A_{ev}^2$		<code>\dynkin A[2]{even}</code>
$A_{od}^2$		<code>\dynkin A[2]{odd}</code>
$D_n^2$		<code>\dynkin D[2]{}</code>
$E_6^2$		<code>\dynkin E[2]6</code>
$D_4^3$		<code>\dynkin D[3]4</code>

Table 13: Some more affine Dynkin diagrams

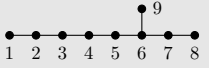
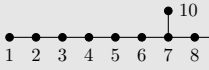
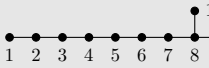
$A_4^2$		<code>\dynkin A[2]4</code>
$A_5^2$		<code>\dynkin A[2]5</code>
$A_6^2$		<code>\dynkin A[2]6</code>
$A_7^2$		<code>\dynkin A[2]7</code>
$A_8^2$		<code>\dynkin A[2]8</code>
$D_3^2$		<code>\dynkin D[2]3</code>
$D_4^2$		<code>\dynkin D[2]4</code>
$D_5^2$		<code>\dynkin D[2]5</code>
$D_6^2$		<code>\dynkin D[2]6</code>
$D_7^2$		<code>\dynkin D[2]7</code>
$D_8^2$		<code>\dynkin D[2]8</code>
$D_4^3$		<code>\dynkin D[3]4</code>
$E_6^2$		<code>\dynkin E[2]6</code>

Table 14: Some more Kac–Moody Dynkin diagrams, only allowed in Kac ordering

$E_6$		<code>\dynkin [ordering=Kac,label]E6</code>
$E_7$		<code>\dynkin [ordering=Kac,label]E7</code>
$E_8$		<code>\dynkin [ordering=Kac,label]E8</code>

continued ...

Table 14: ...continued

$E_9$		<code>\dynkin [ordering=Kac,label]E9</code>
$E_{10}$		<code>\dynkin [ordering=Kac,label]E{10}</code>
$E_{11}$		<code>\dynkin [ordering=Kac,label]E{11}</code>

## 24. EXTENDED COXETER DIAGRAMS

Extended and Coxeter options together

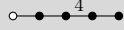
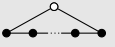

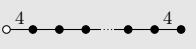
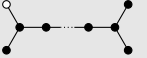
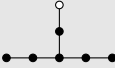
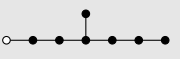
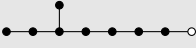
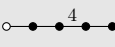


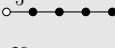

`\dynkin[extended,Coxeter]F4`

Table 15: The extended (affine) Coxeter diagrams

$A_n$		<code>\dynkin [extended,Coxeter]A{}</code>
$B_n$		<code>\dynkin [extended,Coxeter]B{}</code>
$C_n$		<code>\dynkin [extended,Coxeter]C{}</code>
$D_n$		<code>\dynkin [extended,Coxeter]D{}</code>
$E_6$		<code>\dynkin [extended,Coxeter]E6</code>
$E_7$		<code>\dynkin [extended,Coxeter]E7</code>
$E_8$		<code>\dynkin [extended,Coxeter]E8</code>
$F_4$		<code>\dynkin [extended,Coxeter]F4</code>
$G_2$		<code>\dynkin [extended,Coxeter]G2</code>
$H_3$		<code>\dynkin [extended,Coxeter]H3</code>
$H_4$		<code>\dynkin [extended,Coxeter]H4</code>
$I_1$		<code>\dynkin [extended,Coxeter]I1</code>

## 25. KAC STYLE

We include a style called `Kac` which tries to imitate the style of [16].

Kac style

`\dynkin[Kac]F4`

$\circ - \circ \Rightarrow \circ - \circ$

Table 16: The Dynkin diagrams of the simple root systems in Kac style

$A_n$	$\circ - \circ - \cdots - \circ - \circ$	<code>\dynkin A{}</code>
$B_n$	$\circ - \circ - \cdots - \circ - \circ \Rightarrow \circ$	<code>\dynkin B{}</code>
$C_n$	$\circ - \circ - \cdots - \circ - \circ \Leftarrow \circ$	<code>\dynkin C{}</code>
$D_n$	$\circ - \circ - \cdots - \circ - \circ \begin{array}{l} \nearrow \circ \\ \searrow \circ \end{array}$	<code>\dynkin D{}</code>
$E_6$	$\begin{array}{c} \circ \\   \\ \circ - \circ - \cdots - \circ - \circ \end{array}$	<code>\dynkin E6</code>
$E_7$	$\begin{array}{c} \circ \\   \\ \circ - \circ - \cdots - \circ - \circ - \circ \end{array}$	<code>\dynkin E7</code>
$E_8$	$\begin{array}{c} \circ \\   \\ \circ - \circ - \cdots - \circ - \circ - \circ - \circ \end{array}$	<code>\dynkin E8</code>
$F_4$	$\circ - \circ \Rightarrow \circ - \circ$	<code>\dynkin F4</code>
$G_2$	$\circ \Rightarrow \circ$	<code>\dynkin G2</code>

Table 17: The Dynkin diagrams of the extended simple root systems in Kac style

$A_1^1$	$\circ \rightleftharpoons \circ$	<code>\dynkin [extended] A1</code>
$A_n^1$	$\begin{array}{c} \circ \\ \diagup \quad \diagdown \\ \circ - \circ - \cdots - \circ - \circ \end{array}$	<code>\dynkin [extended] A{}</code>
$B_n^1$	$\begin{array}{c} \circ \\   \\ \circ - \circ - \cdots - \circ - \circ \Rightarrow \circ \\ \diagup \quad \diagdown \\ \circ \quad \circ \end{array}$	<code>\dynkin [extended] B{}</code>
$C_n^1$	$\circ \Rightarrow \circ - \circ - \cdots - \circ - \circ \Leftarrow \circ$	<code>\dynkin [extended] C{}</code>

continued ...

Table 17: ...continued

$D_n^1$		<code>\dynkin [extended] D{}</code>
$E_6^1$		<code>\dynkin [extended] E6</code>
$E_7^1$		<code>\dynkin [extended] E7</code>
$E_8^1$		<code>\dynkin [extended] E8</code>
$F_4^1$	$\circ - \circ - \circ \Rightarrow \circ - \circ$	<code>\dynkin [extended] F4</code>
$G_2^1$	$\circ - \circ \Rightarrow \circ$	<code>\dynkin [extended] G2</code>

Table 18: The Dynkin diagrams of the twisted simple root systems in Kac style

$A_2^2$	$\circ \Leftarrow \circ$	<code>\dynkin [extended] A[2] 2</code>
$A_{ev}^2$	$\circ \Leftarrow \circ - \circ - \circ - \dots - \circ - \circ \Leftarrow \circ$	<code>\dynkin [extended] A[2] {even}</code>
$A_{od}^2$	$\circ - \circ - \dots - \circ - \circ \Leftarrow \circ$	<code>\dynkin [extended] A[2] {odd}</code>
$D_n^2$	$\circ \Leftarrow \circ - \circ - \dots - \circ - \circ - \circ \Rightarrow \circ$	<code>\dynkin [extended] D[2] {}</code>
$E_6^2$	$\circ - \circ - \circ \Leftarrow \circ - \circ$	<code>\dynkin [extended] E[2] 6</code>
$D_4^3$	$\circ - \circ \Leftarrow \circ$	<code>\dynkin [extended] D[3] 4</code>

## 26. CEREF STYLE

We include a style called **ceref** which paints oblong root markers with shadows. The word “ceref” is an old form of the word “serif”.

Ceref style

`\dynkin[ceref] F4`

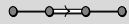


Table 19: The Dynkin diagrams of the simple root systems in cref style


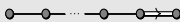

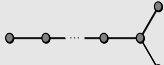



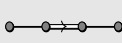

$A_n$		<code>\dynkin A{}</code>
$B_n$		<code>\dynkin B{}</code>
$C_n$		<code>\dynkin C{}</code>
$D_n$		<code>\dynkin D{}</code>
$E_6$		<code>\dynkin E6</code>
$E_7$		<code>\dynkin E7</code>
$E_8$		<code>\dynkin E8</code>
$F_4$		<code>\dynkin F4</code>
$G_2$		<code>\dynkin G2</code>

Table 20: The Dynkin diagrams of the extended simple root systems in cref style


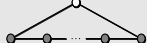
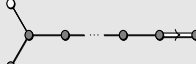

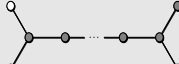

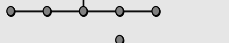
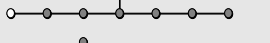
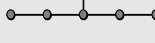

$A_1^1$		<code>\dynkin [extended] A1</code>
$A_n^1$		<code>\dynkin [extended] A{}</code>
$B_n^1$		<code>\dynkin [extended] B{}</code>
$C_n^1$		<code>\dynkin [extended] C{}</code>
$D_n^1$		<code>\dynkin [extended] D{}</code>
$E_6^1$		<code>\dynkin [extended] E6</code>
$E_7^1$		<code>\dynkin [extended] E7</code>
$E_8^1$		<code>\dynkin [extended] E8</code>
$F_4^1$		<code>\dynkin [extended] F4</code>
$G_2^1$		<code>\dynkin [extended] G2</code>

Table 21: The Dynkin diagrams of the twisted simple root systems in ceref style

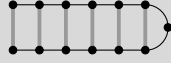
$A_2^2$		<code>\dynkin [extended] A[2] 2</code>
$A_{ev}^2$		<code>\dynkin [extended] A[2] {even}</code>
$A_{od}^2$		<code>\dynkin [extended] A[2] {odd}</code>
$D_n^2$		<code>\dynkin [extended] D[2] {}</code>
$E_6^2$		<code>\dynkin [extended] E[2] 6</code>
$D_4^3$		<code>\dynkin [extended] D[3] 4</code>

## 27. MORE ON FOLDED DYNKIN DIAGRAMS

The Dynkin diagrams package has limited support for folding Dynkin diagrams.

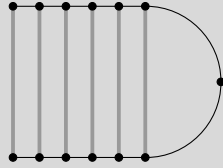
### Folding

```
\dynkin[fold]A{13}
```



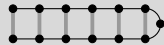
### Big fold radius

```
\dynkin[fold,fold radius=1cm]A{13}
```



### Small fold radius

```
\dynkin[fold,fold radius=.2cm]A{13}
```

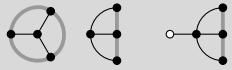


Some Dynkin diagrams have multiple foldings, which we attempt to distinguish (not entirely successfully) by their *ply*: the maximum number of roots folded together. Most diagrams can only allow a 2-ply folding, so `fold` is a synonym for `ply=2`.



3-ply

```
\dynkin[ply=3]D4
\dynkin[ply=3,fold right]D4
\dynkin[ply=3]D[1]4
```



4-ply

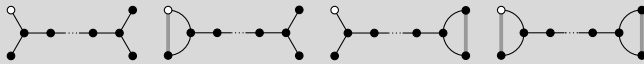
```
\dynkin[ply=4]D[1]4
```



The  $D_\ell^{(1)}$  diagrams can be folded on their left end and separately on their right end:

Left, right and both

```
\dynkin D[1]{ } \
\dynkin[fold left]D[1]{ } \
\dynkin[fold right]D[1]{ } \
\dynkin[fold]D[1]{ }
```



We have to be careful about the 4-ply foldings of  $D_{2\ell}^{(1)}$ , for which we can have two different patterns, so by default, the package only draws as much as it can without distinguishing the two:

Default  $D_{2\ell}^{(1)}$  and the two ways to finish it

```
\dynkin[ply=4]D[1]{****.*****.*****} \
\begin{dynkinDiagram}[ply=4]{D}[1]{****.*****.*****}
  \dynkinFold[bend right=90]1{13}
  \dynkinFold[bend right=90]0{14}
\end{dynkinDiagram} \
\begin{dynkinDiagram}[ply=4]{D}[1]{****.*****.*****}
  \dynkinFold01
  \dynkinFold1{13}
  \dynkinFold{13}{14}
\end{dynkinDiagram}
```

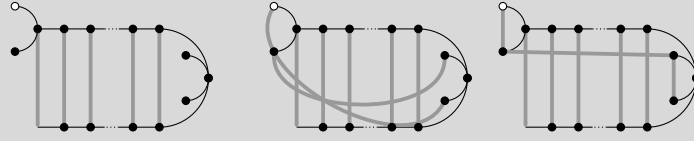


Table 22: Some foldings of Dynkin diagrams. For these diagrams, we want to compare a folding diagram with the diagram that results when we fold it, so it looks best to set `fold radius` and `edge length` to equal lengths.

$A_3$		<code>\dynkin [fold]A[0]3</code>
$C_2$		<code>\dynkin C[0]2</code>
$A_{2\ell-1}$		<code>\dynkin [fold]A{**.*****.**}</code>
$C_\ell$		<code>\dynkin C{}</code>
$B_3$		<code>\dynkin [fold]B[0]3</code>
$G_2$		<code>\dynkin [reverse arrows]G[0]2</code>
$D_4$		<code>\dynkin [ply=3,fold right]D4</code>
$G_2$		<code>\dynkin G2</code>

continued ...

Table 22: ...continued

$D_{\ell+1}$		<code>\dynkin [fold]D{}</code>
$B_{\ell}$		<code>\dynkin B{}</code>
$E_6$		<code>\dynkin [fold]E[0]6</code>
$F_4$		<code>\dynkin [reverse arrows]F[0]4</code>
$A_3^1$		<code>\dynkin [ply=4]A[1]3</code>
$A_1^1$		<code>\dynkin A[1]1</code>
$A_{2\ell-1}^1$		<code>\dynkin [fold]A[1]{**.*.....**}</code>
$C_{\ell}^1$		<code>\dynkin C[1]{}</code>
$B_3^1$		<code>\dynkin [ply=3]B[1]3</code>
$A_2^2$		<code>\dynkin A[2]2</code>
$B_3^1$		<code>\dynkin [ply=2]B[1]3</code>
$G_2^1$		<code>\dynkin G[1]2</code>
$B_{\ell}^1$		<code>\dynkin [fold]B[1]{}</code>
$D_{\ell}^2$		<code>\dynkin D[2]{}</code>
$D_4^1$		<code>\dynkin [ply=3]D[1]4</code>
$B_3^1$		<code>\dynkin B[1]3</code>
$D_4^1$		<code>\dynkin [ply=3]D[1]4</code>
$G_2^1$		<code>\dynkin G[1]2</code>
$D_{\ell+1}^1$		<code>\dynkin [fold]D[1]{}</code>
$D_{\ell}^2$		<code>\dynkin D[2]{}</code>
$D_{\ell+1}^1$		<code>\dynkin [fold right]D[1]{}</code>
$B_{\ell}^1$		<code>\dynkin B[1]{}</code>

continued ...

Table 22: ...continued

$D_{2\ell}^1$		<pre>\begin{dynkinDiagram}[ply=4]D[1]% {****.*****.*****} \dynkinFold01 \dynkinFold1{13} \dynkinFold{13}{14} \end{dynkinDiagram}</pre>
$A_{\text{odd}}^2$		<pre>\dynkin A[2]{odd}</pre>
$D_{2\ell}^1$		<pre>\begin{dynkinDiagram}[ply=4]{D}[1]% {****.*****.*****} \dynkinFold[bend right=90]1{13} \dynkinFold[bend right=90]0{14} \end{dynkinDiagram}</pre>
$A_{\text{even}}^2$		<pre>\dynkin A[2]{even}</pre>
$E_6^1$		<pre>\dynkin [fold]E[1]6</pre>
$F_4^1$		<pre>\dynkin [reverse arrows]F[1]4</pre>
$E_6^1$		<pre>\dynkin [ply=3]E[1]6</pre>
$D_4^3$		<pre>\dynkin D[3]4</pre>
$E_7^1$		<pre>\dynkin [fold]E[1]7</pre>
$E_6^2$		<pre>\dynkin E[2]6</pre>
$F_4^1$		<pre>\dynkin [fold]F[1]4</pre>
$G_2^1$		<pre>\dynkin G[1]2</pre>
$A_{\text{odd}}^2$		<pre>\dynkin [odd,fold]A[2]{****.***}</pre>
$A_{\text{even}}^2$		<pre>\dynkin A[2]{even}</pre>
$D_3^2$		<pre>\dynkin [fold]D[2]3</pre>
$A_2^2$		<pre>\dynkin A[2]2</pre>

Table 23: Frobenius fixed point subgroups of finite simple groups of Lie type [4] p. 15

$A_{\ell \geq 1}$		<code>\dynkin A{}</code>
${}^2A_{\ell \geq 2}$		<code>\dynkin [fold]A{}</code>
$B_{\ell \geq 2}$		<code>\dynkin B{}</code>
${}^2B_2$		<code>\dynkin [fold]B2</code>
$C_{\ell \geq 3}$		<code>\dynkin C{}</code>
$D_{\ell \geq 4}$		<code>\dynkin D{}</code>
${}^2D_{\ell \geq 4}$		<code>\dynkin [fold]D{}</code>
${}^3D_4$		<code>\dynkin [ply=3]D4</code>
$E_6$		<code>\dynkin E6</code>
${}^2E_6$		<code>\dynkin [fold]E6</code>
$E_7$		<code>\dynkin E7</code>
$E_8$		<code>\dynkin E8</code>
$F_4$		<code>\dynkin F4</code>
${}^2F_4$		<code>\dynkin [fold]F4</code>
$G_2$		<code>\dynkin G2</code>
${}^2G_2$		<code>\dynkin [fold]G2</code>

## 28. TYPESETTING MATHEMATICAL NAMES OF DYNKIN DIAGRAMS

The `\dynkinName` command, with the same syntax as `\dynkin`, typesets a default name of your diagram in L<sup>A</sup>T<sub>E</sub>X. It is perhaps only useful when automatically generating a large collection of Dynkin diagrams in a computer program.

Name of a diagram

```
\dynkinName[label,extended]B7
\dynkinName A[2]{even}
\dynkinName[Coxeter]B7
\dynkinName[label,extended]B{}
\dynkinName D[3]4
```

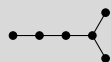
$B_7^1$   $A_{ev}^2$   $B_7$   $B_n^1$   $D_4^3$

## 29. CONNECTING DYNKIN DIAGRAMS

We can make some sophisticated folded diagrams by drawing multiple diagrams, each with a name:

Name a diagram

```
\dynkin[name=Bob]D6
```



We can then connect the two with folding edges:

Connect diagrams

```
\begin{dynkinDiagram}[name=upper]A3
  \node (current) at ($ (upper root 1)+(0,-.3cm)$) {};
  \dynkin[at=(current),name=lower]A3
  \begin{pgfonlayer}{Dynkin behind}
    \foreach \i in {1,...,3}%
    {%
      \draw[/Dynkin diagram/fold style]
        ($ (upper root \i)$)
        -- ($ (lower root \i)$);%
    }%
  \end{pgfonlayer}
\end{dynkinDiagram}
```



The nonsplit Freudenthal–Tits magic square

```
\newcommand\clrK{\rowcolor{BurntOrange!80}}
\newcommand\clrL{\rowcolor{SeaGreen}}
\newcommand\clrH{\rowcolor{RoyalBlue!50}}
\newcommand\clrO{\rowcolor{OrangeRed!70}}
\newcommand\clrOO{\cellcolor{Red}}
\NewDocumentCommand\hd{om}{
  \cellcolor{gray!30}$\IfNoValueF{#1}{\mathbb{#1}}\setminus\mathbb{#2}$}
\tikzset{/Dynkin diagram/fold style/.style={blue!22,ultra thick}}
\begin{tcolorbox}[colback=white,colframe=white]
\begin{tabular}{|c|c|c|c|c|c|}\hline
\hd[A]{B}&\hd{K}&\hd{L}&\hd{H}&\hd{O}&\hd{O}\\\hline
\clrK\hd{K}&\dynkin A1 & \dynkin A{*o} & \dynkin C{o*o} & \dynkin
F{*ooo} & \hd{L}& \dynkin A{**} &
\end{tabular}
\end{tcolorbox}
```

```

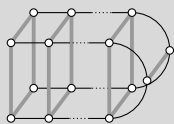
\begin{dynkinDiagram}[name=upper]A2
\node (current) at ($(upper root 1)+(0,-.35cm)$) {};
\dynkin[at=(current),name=lower]A2
\begin{pgfonlayer}{Dynkin behind}
\foreach \i in {1,2}{%
\draw[/Dynkin diagram/fold style] ($(upper root \i)$) -- ($(lower
root \i)$);}
\end{pgfonlayer}
\end{dynkinDiagram}&
\dynkin A{*ooo*} &
\dynkin E{*oooo*} \\ \hline
\clrH\hd{H} &
\dynkin C{***} &
\dynkin[fold] A{*****} &
\dynkin D{*oo*o*} &
\dynkin E{*oooo**}\\ \hline
\clrO\hd{O} &
\dynkin F{****} &
\dynkin[o/.style = {
solid,
draw=black,
fill=black}] E{II} &
\dynkin[backwards] E{*o**oo*o} &
\clrOO \dynkin E{*oooo**}\\ \hline
\end{tabular}
\end{tcolorbox}

```

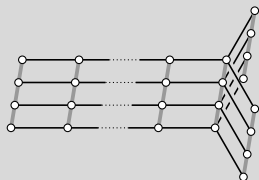
A \ B	K	L	H	O
K				
L				
H				
O				

The following diagrams arise in the Satake diagrams of the pseudo-Riemannian symmetric spaces [2].

```
\pgfkeys{/Dynkin diagram,edge length=.5cm,fold radius=.5cm}
\begin{tikzpicture}
  \dynkin[name=1]A{IIIb}
  \node (a) at (-.3,-.4){};
  \dynkin[name=2,at=(a)]A{IIIb}
  \begin{pgfonlayer}{Dynkin behind}
    \foreach \i in {1,...,7}{
      \draw[/Dynkin diagram/fold style]
        ($ (1 root \i)$) -- ($ (2 root \i)$);}
  \end{pgfonlayer}
\end{tikzpicture}
```

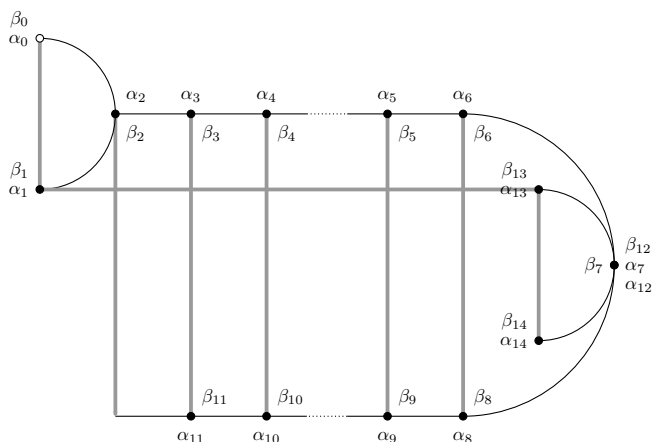


```
\pgfkeys{/Dynkin diagram,
  edge length=.75cm,
  edge/.style={draw=example-color,double=black,very thick}}
\begin{tikzpicture}
  \foreach \d in {1,...,4}{
    \node (current) at ($ (\d*.05,\d*.3)$){};
    \dynkin[name=\d,at=(current)]D{oo.oooo}
  }
  \begin{pgfonlayer}{Dynkin behind}
    \newcommand\df[2]{
      \draw[/Dynkin diagram/fold style]
        ($ (#1 root \i)$) -- ($ (#2 root \i)$);}
    \foreach \i in
      {1,...,6}{\df{1}{2}\df{2}{3}\df{3}{4}}
  \end{pgfonlayer}
\end{tikzpicture}
```





## 30. OTHER EXAMPLES

 $^1D_4$  4-ply tied bending:

```

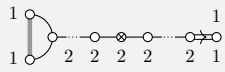
\tikzset{/Dynkin diagram,
  edge length=1cm,
  fold radius=1cm,
  label,
  label*=true,
  label macro/.code={\alpha_{#1}},
  label macro*/.code={\beta_{#1}}}
\({}^1 D_4\) 4-ply tied straight:
\begin{dynkinDiagram}[ply=4]D[1]%
{****.*****.*****}
  \dynkinFold 01
  \dynkinFold 1{13}
  \dynkinFold{13}{14}
\end{dynkinDiagram}
\({}^1 D_4\) 4-ply tied bending:
\begin{dynkinDiagram}[ply=4,label]D[1]%
{****.*****.*****}
  \dynkinFold1{13}
  \dynkinFold[bend right=65]0{14}
\end{dynkinDiagram}

```

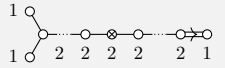
Below we draw the Vogan diagrams of some affine Lie superalgebras [22, 21].

$\mathfrak{sl}(2m|2n)^{(2)}$

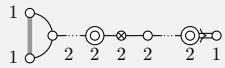
```
\begin{dynkinDiagram}[ply=2,label]{B}[1]{oo.oto.oo}
  \dynkinLabelRoot*71
\end{dynkinDiagram}
```



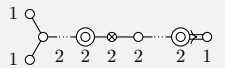
```
\dynkin[label]B[1]{oo.oto.oo}
```



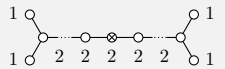
```
\dynkin[ply=2,label]B[1]{oo.0to.0o}
```



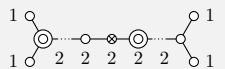
```
\dynkin[label]B[1]{oo.0to.0o}
```

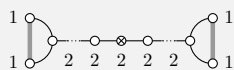


```
\dynkin[label]D[1]{oo.oto.ooo}
```

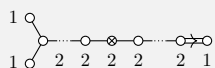


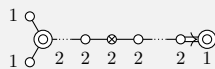
```
\dynkin[label]D[1]{o0.ot0.ooo}
```

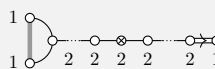


$$\backslash\text{dynkin}[\text{label},\text{fold}]\text{D}[1]\{\text{oo.oto.ooo}\}$$


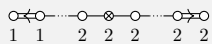
$$\mathfrak{sl}(2m+1|2n)^2$$

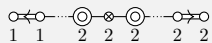
$$\backslash\text{dynkin}[\text{label}]\text{B}[1]\{\text{oo.oto.oo}\}$$


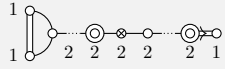
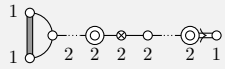
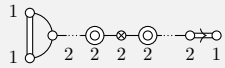
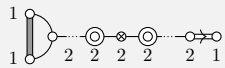
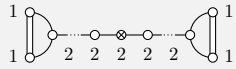
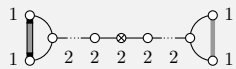
$$\backslash\text{dynkin}[\text{label}]\text{B}[1]\{\text{o0.oto.o0}\}$$


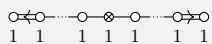
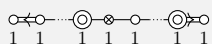
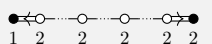
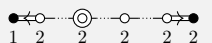
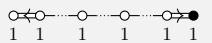
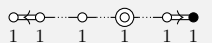
$$\backslash\text{dynkin}[\text{label},\text{fold}]\text{B}[1]\{\text{oo.oto.oo}\}$$


$$\mathfrak{sl}(2m+1|2n+1)^2$$

$$\backslash\text{dynkin}[\text{label}]\text{D}[2]\{\text{o.oto.oo}\}$$


$$\backslash\text{dynkin}[\text{label}]\text{D}[2]\{\text{o.0t0.oo}\}$$


$\mathfrak{sl}(2|2n+1)^{(2)}$ 
`\dynkin[ply=2,label,double edges]B[1]{oo.0to.0o}`

`\dynkin[ply=2,label,double fold]B[1]{oo.0to.0o}`

`\dynkin[ply=2,label,double edges]B[1]{oo.0t0.oo}`

`\dynkin[ply=2,label,double fold]B[1]{oo.0t0.oo}`

 $\mathfrak{sl}(2|2n)^{(2)}$ 
`\dynkin[ply=2,label,double edges]D[1]{oo.oto.ooo}`

`\dynkin[ply=2,label,double fold left]D[1]{oo.oto.ooo}`


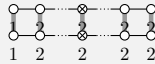
$\mathfrak{osp}(2m|2n)^{(2)}$ 
`\dynkin[label,label macro/.code={1}]D[2]{o.oto.oo}`

`\dynkin[label,label macro/.code={1}]D[2]{o.0to.0o}`

 $\mathfrak{osp}(2|2n)^{(2)}$ 
`\dynkin[label,label macro/.code=\lablIt{#1},  
affine mark=*]  
D[2]{o.o.o.o*}`

`\dynkin[label,label macro/.code=\lablIt{#1},  
affine mark=*]  
D[2]{o.0.o.o*}`

 $\mathfrak{sl}(1|2n+1)^4$ 
`\dynkin[label,label macro/.code={1}]D[2]{o.o.o.o*}`

`\dynkin[label,label macro/.code={1}]D[2]{o.o.0.o*}`


$A^1$ 

```

\begin{tikzpicture}
  \dynkin[name=upper]A{oo.t.oo}
  \node (Dynkin current) at (upper root 1){};
  \dynkinSouth
  \dynkin[at=(Dynkin
current),name=lower]A{oo.t.oo}
  \begin{pgfonlayer}{Dynkin behind}
  \foreach \i in {1,...,5}{
    \draw[/Dynkin diagram/fold style]
      ($(\upper root \i)$) --
      ($(\lower root \i)$);
  }
  \end{pgfonlayer}
\end{tikzpicture}

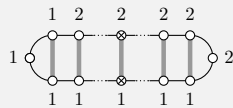
```



```

\dynkin[fold]A[1]{oo.t.oooo.t.oo}

```



```

\dynkin[fold,affine mark=t]A[1]{oo.o.ootoo.o.oo}

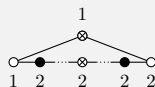
```

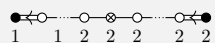
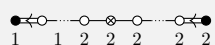
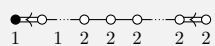
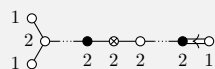
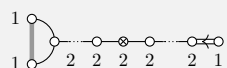
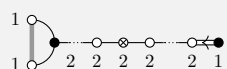


```

\dynkin[affine mark=t]A[1]{o*.t.*o}

```



$B^1$ `\dynkin[affine mark=*]A[2]{o.oto.o*}``\dynkin[affine mark=*]A[2]{o.oto.o*}``\dynkin[affine mark=*]A[2]{o.ooo.oo}``\dynkin[odd]A[2]{oo.*to.*o}``\dynkin[odd,fold]A[2]{oo.oto.oo}``\dynkin[odd,fold]A[2]{o*.oto.o*}`

$D^1$ 

```
\dynkin D{otoo}
```



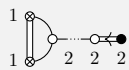
```
\dynkin D{ot*o}
```



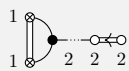
```
\dynkin[fold]D{otoo}
```

 $C^1$ 

```
\dynkin[double edges,fold,affine
mark=t,odd]A[2]{to.o*}
```



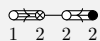
```
\dynkin[double edges,fold,affine
mark=t,odd]A[2]{t*.oo}
```



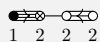


$F^1$ 

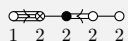
```
\begin{dynkinDiagram}A{oto*}%
    \dynkinQuadrupleEdge 12%
    \dynkinTripleEdge 43%
\end{dynkinDiagram}%
```



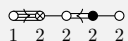
```
\begin{dynkinDiagram}A{*too}%
    \dynkinQuadrupleEdge 12%
    \dynkinTripleEdge 43%
\end{dynkinDiagram}%
```

 $G^1$ 

```
\begin{dynkinDiagram}A{ot*oo}%
    \dynkinQuadrupleEdge 12%
    \dynkinDefiniteDoubleEdge 43%
\end{dynkinDiagram}%
```




```
\begin{dynkinDiagram}A{oto*o}%
    \dynkinQuadrupleEdge 12%
    \dynkinDefiniteDoubleEdge 43%
\end{dynkinDiagram}%
```



<pre> \begin{dynkinDiagram}A{*too*}%     \dynkinQuadrupleEdge 12%     \dynkinDefiniteDoubleEdge 43% \end{dynkinDiagram}% </pre>
<pre> \begin{dynkinDiagram}A{*tooo}%     \dynkinQuadrupleEdge 12%     \dynkinDefiniteDoubleEdge 43% \end{dynkinDiagram}% </pre>

## 31. EXAMPLE: THE COMPLEX SIMPLE LIE ALGEBRAS

$\mathfrak{g}$	Diagram	Weights	Roots	Simple roots
$A_n$		$\frac{1}{n+1}\mathbb{Z}^{n+1}/\langle\sum e_j\rangle$	$e_i - e_j$	$e_i - e_{i+1}$
$B_n$		$\frac{1}{2}\mathbb{Z}^n$	$\pm e_i, \pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, e_n$
$C_n$		$\mathbb{Z}^n$	$\pm 2e_i, \pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, 2e_n$
$D_n$		$\frac{1}{2}\mathbb{Z}^n$	$\pm e_i \pm e_j, i \neq j$	$e_i - e_{i+1}, \quad i \leq n-1$ $e_{n-1} + e_n$
$E_8$		$\frac{1}{2}\mathbb{Z}^8$	$\pm 2e_i \pm 2e_j, \quad i \neq j,$ $\sum_i (-1)^{m_i} e_i, \quad \sum m_i \text{ even}$	$2e_1 - 2e_2,$ $2e_2 - 2e_3,$ $2e_3 - 2e_4,$ $2e_4 - 2e_5,$ $2e_5 - 2e_6,$ $2e_6 + 2e_7,$ $-\sum e_j,$ $2e_6 - 2e_7$
$E_7$		$\frac{1}{2}\mathbb{Z}^8/\langle e_1 - e_2 \rangle$	quotient of $E_8$	quotient of $E_8$
$E_6$		$\frac{1}{3}\mathbb{Z}^8/\langle e_1 - e_2, e_2 - e_3 \rangle$	quotient of $E_8$	quotient of $E_8$
$F_4$		$\mathbb{Z}^4$	$\pm 2e_i,$ $\pm 2e_i \pm 2e_j, \quad i \neq j,$ $\pm e_1 \pm e_2 \pm e_3 \pm e_4$	$2e_2 - 2e_3,$ $2e_3 - 2e_4,$ $2e_4,$ $e_1 - e_2 - e_3 - e_4$

g	Diagram	Weights	Roots	Simple roots
$G_2$		$\mathbb{Z}^3 / \langle \sum e_j \rangle$	$\pm(1, -1, 0),$ $\pm(-1, 0, 1),$ $\pm(0, -1, 1),$ $\pm(2, -1, -1),$ $\pm(1, -2, 1),$ $\pm(-1, -1, 2)$	$(-1, 0, 1),$ $(2, -1, -1)$

```

\NewDocumentEnvironment{bunch}{}{
  \renewcommand*{\arraystretch}{1}
  \begin{array}{@{}ll@{}}
    \\\ \midrule
  }{
    \\\ \midrule\end{array}}
\small
\NewDocumentCommand\nct{mm}{
  \newcolumnntype{#1}{>{\columncolor[gray]{.9}}>{\$}m{#2cm}<{\$}}
\nct{G}{.3}\nct{J}{2.1}\nct{K}{3}\nct{R}{3.7}\nct{S}{3}
\NewDocumentCommand\LieG{}{\mathfrak{g}}
\NewDocumentCommand\W{om}{
  \ensuremath{
    \mathbb{Z}^{\#2}
    \IfValueT{#1}{/\left<#1\right>}}
\renewcommand*{\arraystretch}{1.5}
\NewDocumentCommand\quo{}{\text{quotient of } E_8}
\begin{longtable}{@{}GJKRS@{}}
\LieG&
  \text{Diagram}&
  \text{Weights}&
  \text{Roots}&
  \text{Simple roots}\\
\midrule\endfirsthead
\LieG&
  \text{Diagram}&
  \text{Weights}&
  \text{Roots}&
  \text{Simple roots}\\
\midrule\endhead
A_n&
  \dynkin A{}&
  \frac{1}{n+1}\W[\sum e_j]{n+1}&
  e_i-e_j&
  e_i-e_{i+1}\\
B_n&
  \dynkin B{}&
  \frac{1}{2}\W n&
  \pm e_i, \pm e_i \pm e_j, i \ne j&
  e_i-e_{i+1}, e_n\\
C_n&
  \dynkin C{}&

```

```

\W n&
\pm 2 e_i, \pm e_i \pm e_j, i \ne j&
e_i-e_{i+1}, 2e_n\\
D_n&
\dynkin D{}&
\frac{1}{2}\W n&
\pm e_i \pm e_j, i \ne j &
\begin{bunch}
e_i-e_{i+1}, i \leq n-1\\
e_{n-1}+e_n
\end{bunch}\\
E_8&
\dynkin E8&
\frac{1}{2}\W 8&
\begin{bunch}
\pm 2e_i \pm 2e_j, i \ne j, \\
\sum_i (-1)^{m_i} e_i, \sum m_i \text{ even}
\end{bunch}&
\begin{bunch}
2e_1-2e_2, \\
2e_2-2e_3, \\
2e_3-2e_4, \\
2e_4-2e_5, \\
2e_5-2e_6, \\
2e_6+2e_7, \\
-\sum e_j, 2e_6-2e_7
\end{bunch}\\
E_7&
\dynkin E7&
\frac{1}{2}\W[e_1-e_2]8&
\text{quo}&
\text{quo}\\
E_6&
\dynkin E6&
\frac{1}{3}\W[e_1-e_2, e_2-e_3]8&
\text{quo}&
\text{quo}\\
F_4&
\dynkin F4&
\W4&
\begin{bunch}
\pm 2e_i, \\
\pm 2e_i \pm 2e_j, \text{quad } i \ne j, \\
\pm e_1 \pm e_2 \pm e_3 \pm e_4
\end{bunch}&
\begin{bunch}
2e_2-2e_3, \\
2e_3-2e_4, \\
2e_4, \\
e_1-e_2-e_3-e_4
\end{bunch}\\
G_2&
\dynkin G2&

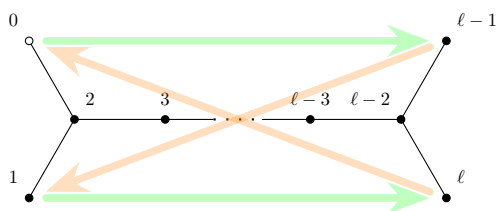
```

```

\W[\sum e_j]3&
\begin{bunch}
  \pm(1,-1,0),\\
  \pm(-1,0,1),\\
  \pm(0,-1,1),\\
  \pm(2,-1,-1),\\
  \pm(1,-2,1),\\
  \pm(-1,-1,2)
\end{bunch}
&
\begin{bunch}
  (-1,0,1),\\
  (2,-1,-1)
\end{bunch}
\end{longtable}

```

## 32. AN EXAMPLE OF MIKHAIL BOROVoi

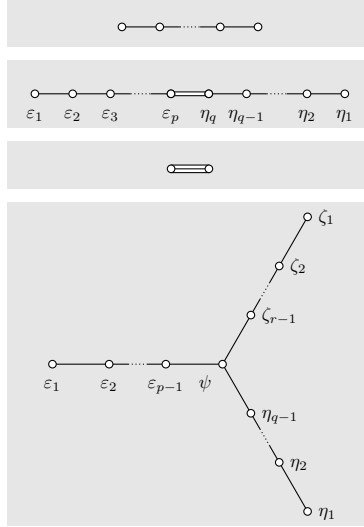


```

\tikzset{
  big arrow/.style={
    -Stealth,
    line cap=round,
    line width=1mm,
    shorten <=1mm,
    shorten >=1mm}
}
\newcommand\catholic[2]{
  \draw[big arrow,green!25!white] (root #1) to (root #2);
}
\newcommand\protestant[2]{
  \begin{scope}[transparency group, opacity=.25]
    \draw[big arrow,orange] (root #1) to (root #2);
  \end{scope}
}
\begin{dynkinDiagram}[%
  edge length=1.2cm,
  indefinite edge/.style={
    thick,
    loosely dotted
  },
  labels*={0,1,2,3,\ell-3,\ell-2,\ell-1,\ell}]
D[1]{}
\catholic 06\catholic 17
\protestant 70\protestant 61
\end{dynkinDiagram}

```

There are many undocumented features, which are not usually very useful; here is a taste, from [14] p. 61.



```

\begin{center}
\makeatletter
\newcommand{\extraNode}[6]{%
{%
\dynkinPlaceRootRelativeTo{#1}{#2}{#3}{#4}{#5}
\dynkinDefiniteSingleEdge{#1}{#2}
\dynkinRootMark{o}{#1}
\advance\dynkin@nodes by 1
\dynkinLabelRoot{#1}{#6}
}%
\newcommand{\extraDotNode}[6]{%
{%
\dynkinPlaceRootRelativeTo{#1}{#2}{#3}{#4}{#5}
\dynkinIndefiniteSingleEdge{#1}{#2}
\dynkinRootMark{o}{#1}
\advance\dynkin@nodes by 1
\dynkinLabelRoot{#1}{#6}
}%
\makeatother
\tikzset{Dynkin diagram,mark=o,edge length=.5cm}
\begin{tabular}{>{\columncolor[gray]{.9}}c}
\dynkin A{}
\\ \midrule
\begin{dynkinDiagram}A{ooo.o}
\dynkinLabelRoot{1}{\varepsilon_1}
\dynkinLabelRoot{2}{\varepsilon_2}
\dynkinLabelRoot{3}{\varepsilon_3}
\dynkinLabelRoot{4}{\varepsilon_p}
\dynkin[at=(root 4),arrows=false]B2
\dynkin[at=(root 2),labels={\eta_q,\eta_{q-1},\eta_2,\eta_1}]A{oo.oo}
\end{dynkinDiagram}
\\ \midrule

```

```

\dynkin[arrows=false] G{2}
\\ \midrule
\begin{dynkinDiagram}[%
labels={\varepsilon_{p-1},\psi,\zeta_{r-1},\eta_{q-1}},
mark=o,edge length=.75cm]D4
\extraDotNode{5}{3}{northeast}{right}{left}{\zeta_2}
\extraDotNode{6}{4}{southeast}{right}{left}{\eta_2}
\extraDotNode{7}{1}{west}{below}{above}{\varepsilon_2}
\extraNode{8}{5}{northeast}{right}{left}{\zeta_1}
\extraNode{9}{6}{southeast}{right}{left}{\eta_1}
\extraNode{10}{7}{west}{below}{above}{\varepsilon_1}
\end{dynkinDiagram}
\end{tabular}
\end{center}

```

### 33. SYNTAX

The syntax is `\dynkin[<options>]{<letter>}{<twisted rank>}{<rank>}` where `<letter>` is A, B, C, D, E, F or G, the family of root system for the Dynkin diagram, `<twisted rank>` is 0, 1, 2, 3 (default is 0) representing:

- 0 finite root system
- 1 affine extended root system, i.e. of type <sup>(1)</sup>
- 2 affine twisted root system of type <sup>(2)</sup>
- 3 affine twisted root system of type <sup>(3)</sup>

and `<rank>` is

- (1) an integer representing the rank or
- (2) blank to represent an indefinite rank or
- (3) the name of a Satake diagram as in section 6.

The environment syntax is `\begin{dynkinDiagram}` followed by the same parameters as `\dynkin`, then various Dynkin diagram and TikZ commands, and then `\end{dynkinDiagram}`.

### 34. OPTIONS

```

*/.style = TikZ style data,
default : solid,draw=black,fill=black
          style for roots like •
o/.style = TikZ style data,
default : solid,draw=black,fill=white
          style for roots like ○
O/.style = TikZ style data,
default : solid,draw=black,fill=white
          style for roots like ⊙
t/.style = TikZ style data,
default : solid,draw=black,fill=black
          style for roots like ⊗
x/.style = TikZ style data,
default : solid,draw=black,line cap=round
          continued ...

```

Table 25: ...continued

style for roots like  $\times$   
`X/.style = TikZ style data,`  
 default : `solid,draw=black,thick,line cap=round`  
 style for roots like  $\times$   
`affine mark = o,O,t,x,X,*,`  
 default : `*`  
 default root mark for root zero in an affine Dynkin diagram  
`arrow shape/.style = TikZ style data,`  
 default : `Computer Modern Rightarrow[black]}`  
 shape of arrow heads for most Dynkin diagrams that have arrows  
`arrow style = TikZ style data,`  
 default : `black`  
 set to override the default style for the arrows in nonsimply laced  
 Dynkin diagrams, including length, width, line width and color  
`arrow width = length,`  
 default : `1.5(root radius)`  
 if you change arrow style or shape, use `arrow width` to say how  
 wide your arrows will be  
`arrows = true or false,`  
 default : `true`  
 whether to draw the arrows that arise along the edges  
`backwards = true or false,`  
 default : `false`  
 whether to reverse right to left  
`bird arrow = true or false,`  
 default : `false`  
 whether to use bird style arrows in  $G_2, F_4$ .  
`Bourbaki arrow = true or false,`  
 default : `false`  
 whether to use Bourbaki style arrows in  $G_2, F_4$ .  
`ceref = true or false,`  
 default : `false`  
 whether to draw roots in a “ceref” style  
`Coxeter = true or false,`  
 default : `false`  
 whether to draw a Coxeter diagram, rather than a Dynkin diagram  
`double edges = TikZ style data,`  
 default : not set  
 set to override the `fold` style when folding roots together in a  
 Dynkin diagram, so that the foldings are indicated with double  
 edges (like those of an  $F_4$  Dynkin diagram without arrows)  
`double fold = TikZ style data,`  
 default : not set

continued ...



Table 25: ...continued

set to override the `fold` style when folding roots together in a Dynkin diagram, so that the foldings are indicated with double edges (like those of an  $F_4$  Dynkin diagram without arrows), but filled in solidly

`double left = TikZ style data,`  
`default : not set`  
 set to override the `fold` style when folding roots together at the left side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an  $F_4$  Dynkin diagram without arrows)

`double fold left = TikZ style data,`  
`default : not set`  
 set to override the `fold` style when folding roots together at the left side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an  $F_4$  Dynkin diagram without arrows), but filled in solidly

`double right = TikZ style data,`  
`default : not set`  
 set to override the `fold` style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an  $F_4$  Dynkin diagram without arrows)

`double fold right = TikZ style data,`  
`default : not set`  
 set to override the `fold` style when folding roots together at the right side of a Dynkin diagram, so that the foldings are indicated with double edges (like those of an  $F_4$  Dynkin diagram without arrows), but filled in solidly

`edge label/.style = TikZ style data,`  
`default : text height=0,text depth=0,label distance=-2pt`  
 style of edge labels in the Dynkin diagram, as found, for example, on some Coxeter diagrams

`edge length = length,`  
`default : .35cm`  
 distance between nodes in the Dynkin diagram

`edge/.style = TikZ style data,`  
`default : solid,draw=black,fill=white,thin`  
 style of edges in the Dynkin diagram

`extended = true or false,`  
`default : false`  
 Is this an extended Dynkin diagram?

`fold = true or false,`  
`default : true`  
 whether, when drawing Dynkin diagrams, to draw them 2-ply

`fold left = true or false,`  
`default : true`

continued ...

Table 25: ...continued

whether to fold the roots on the left side of a Dynkin diagram  
**fold radius** = length,  
 default : .3cm  
 the radius of circular arcs used in curved edges of folded Dynkin diagrams  
**fold right** = true or false,  
 default : true  
 whether to fold the roots on the right side of a Dynkin diagram  
**fold left style/.style** = TikZ style data,  
 default :  
 style to override the **fold** style when folding roots together on the left half of a Dynkin diagram  
**fold right style/.style** = TikZ style data,  
 default :  
 style to override the **fold** style when folding roots together on the right half of a Dynkin diagram  
**fold style/.style** = TikZ style data,  
 default : **solid,draw=black!40,fill=none,line width=radius**  
 when drawing folded diagrams, style for the fold indicators  
**gonality** = math,  
 default : 0  
 the gonality of a  $G$  or  $I$  Coxeter diagram  
**horizontal shift** = length,  
 default : 0  
 the gonality of a  $G$  or  $I$  Coxeter diagram  
**indefinite edge ratio** = float,  
 default : 1.6  
 ratio of indefinite edge lengths to other edge lengths  
**indefinite edge/.style** = TikZ style data,  
 default : **solid,draw=black,fill=white,thin,densely dotted**  
 style of the dotted or dashed middle third of each indefinite edge  
**involution/.style** = TikZ style data,  
 default : **latex-latex,black**  
 style of involution arrows  
**involutions** = semicolon separated list of pairs,  
 default :  
 involution double arrows to draw  
**Kac** = true or false,  
 default : false  
 whether to draw in the style of [16]  
**Kac arrows** = true or false,  
 default : false  
 whether to draw arrows in the style of [16]  
**label** = true or false,  
 default : false  
 whether to label the roots according to the current labelling scheme  
 continued ...

Table 25: ...continued

`label* = true` or `false`,  
 default : `false`  
     whether to label the roots at alternative label locations according  
     to the current labelling scheme  
`label depth` = 1-parameter  $\text{\TeX}$  macro,  
 default : `g`  
     the current maximal depth of text labels for the roots, set by  
     giving mathematics text of that depth  
`label directions` = comma separated list,  
 default :  
     list of directions to place root labels: above, below, right, left,  
     below right, and so on.  
`label* directions` = comma separated list,  
 default :  
     list of directions to place alternate root labels: above, below, right,  
     left, below right, and so on.  
`label height` =  $\langle$ 1-parameter  $\text{\TeX}$  macro $\rangle$ ,  
 default : `b`  
     the current maximal height of text labels for the roots, set by  
     giving mathematics text of that height  
`label macro` = 1-parameter  $\text{\TeX}$  macro,  
 default : `#1`  
     the current labelling scheme for roots  
`label macro*` =  $\langle$ 1-parameter  $\text{\TeX}$  macro $\rangle$ ,  
 default : `#1`  
     the current labelling scheme for alternate roots  
`make indefinite edge` =  $\langle$ edge pair  $i$ - $j$  or list of such $\rangle$ ,  
 default : `{}`  
     edge pair or list of edge pairs to treat as having indefinitely many  
     roots on them  
`mark` =  $\langle$ o,0,t,x,X,\* $\rangle$ ,  
 default : `*`  
     default root mark  
`name` =  $\langle$ string $\rangle$ ,  
 default : `anonymous`  
     A name for the Dynkin diagram, with `anonymous` treated as a  
     blank; see section 29  
`ordering` =  $\langle$ Adams, Bourbaki, Carter, Dynkin, Kac $\rangle$ ,  
 default : `Bourbaki`  
     which ordering of the roots to use in exceptional root systems as  
     in section 20  
`parabolic` =  $\langle$ integer $\rangle$ ,  
 default : `0`  
     A parabolic subgroup with specified integer, where the integer  
     is computed as  $n = \sum 2^{i-1}a_i$ ,  $a_i = 0$  or 1, to say that root  $i$  is  
     crossed, i.e. a noncompact root  
     continued ...

Table 25: ...continued

`ply =  $\langle 0, 1, 2, 3, 4 \rangle$ ,`  
`default : 0`  
     how many roots get folded together, at most  
`reverse arrows = true or false,`  
`default : true`  
     whether to reverse the direction of the arrows that arise along the  
     edges  
`root radius =  $\langle \text{number} \rangle$ cm,`  
`default : .05cm`  
     size of the dots and of the crosses in the Dynkin diagram  
`separator length = length,`  
`default : .35cm`  
     distance between successive components of a disconnected Dynkin  
     diagram  
`text style = TikZ style data,`  
`default : scale=.7`  
     Style for any labels on the roots  
`upside down = true or false,`  
`default : false`  
     whether to reverse up to down  
`vertical shift =  $\langle \text{length} \rangle$ ,`  
`default : .5ex`  
     amount to shift up the Dynkin diagram, from the origin of TikZ  
     coordinates.

All other options are passed to TikZ.

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