

The Atomic Theory as Applied To Gases, with Some Experiments on the Viscosity of Air

by

Silas W. Holman

Submitted to the Department of Physics
in partial fulfillment of the requirements for the degree of

BACHELOR OF SCIENCE IN PHYSICS

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 1876

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ABSTRACT

The developments of the “kinetic theory” of gases made within the last ten years have enabled it to account satisfactorily for many of the laws of gases. The mathematical deductions of Clausius, Maxwell and others, based upon the hypothesis of a gas composed of molecules acting upon each other at impact like perfectly elastic spheres, have furnished expressions for the laws of its elasticity, viscosity, conductivity for heat, diffusive power and other properties. For some of these laws we have experimental data of value in testing the validity of these deductions and assumptions. Next to the elasticity, perhaps the phenomena of the viscosity of gases are best adapted to investigation.¹

Thesis supervisor: Edward C. Pickering

Title: Professor of Physics

¹Text from Holman (1876): doi:[10.2307/25138434](https://doi.org/10.2307/25138434).

Acknowledgments

Write your acknowledgments here.

Biographical Sketch

Silas Whitcomb Holman was born in Harvard, Massachusetts on January 20, 1856. He received his S.B. degree in Physics from MIT in 1876, and then joined the MIT Department of Physics as an Assistant. He became Instructor in Physics in 1880, Assistant Professor in 1882, Associate Professor in 1885, and Full Professor in 1893. Throughout this period, he struggled with increasingly severe rheumatoid arthritis. At length, he was defeated, becoming Professor Emeritus in 1897 and dying on April 1, 1900.

Holman's light burned brilliantly before his tragic and untimely death. He published extensively in thermal physics, and authored textbooks on precision measurement, fundamental mechanics, and other subjects. He established the original Heat Measurements Laboratory. Holman was a much admired teacher among both his students and his colleagues. The reports of his department and of the Institute itself refer to him frequently in the 1880's and 1890's, in tones that gradually shift from the greatest respect to the deepest sympathy.

Holman was a student of Professor Edward C. Pickering, then head of the Physics department. Holman himself became second in command of Physics, under Professor Charles R. Cross, some years later. Among Holman's students, several went on to distinguish themselves, including: the astronomer George E. Hale ('90) who organized the Yerkes and Mt. Wilson observatories and who designed the 200 inch telescope on Mt. Palomar; Charles G. Abbot ('94), also an astrophysicist and later Secretary of the Smithsonian Institution; and George K. Burgess ('96), later Director of the Bureau of Standards.

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Chapter 1

Introduction

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetur id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

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1.1 A section discussing the first issue: J/ψ

We begin with some ideas from the literature [6], [7].

$$\frac{\partial}{\partial t} [\rho(e + |\vec{u}|^2/2)] + \nabla \cdot [\rho(h + |\vec{u}|^2/2)\vec{u}] = -\nabla \cdot \vec{q} + \rho\vec{u} \cdot \vec{g} + \frac{\partial}{\partial x_j} (d_{ji}u_i) \quad (1.1)$$

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend

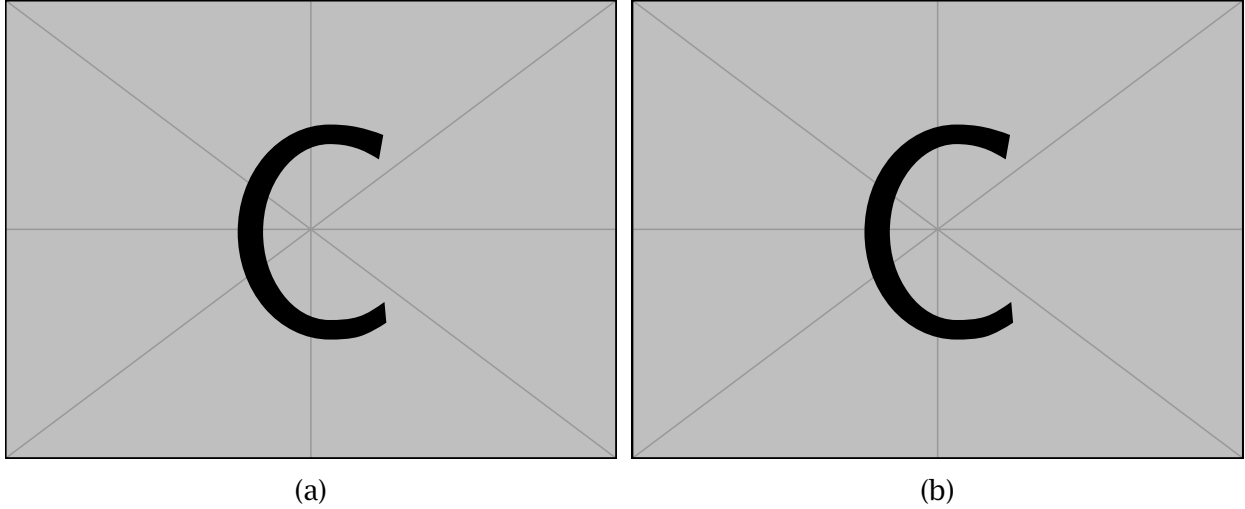


Figure 1.1: A figure with two subfigures: (a) first subfigure; (b) second subfigure.

consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetur adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula. And more citations [7], [8]. Then we write some more and include our citations [9]–[11]. The configuration is shown in Fig. 1.1b.

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetur adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

1.1.1 Subsection eqn. (1.2)

Fusce mauris. Vestibulum luctus nibh at lectus. Sed bibendum, nulla a faucibus semper, leo velit ultricies tellus, ac venenatis arcu wisi vel nisl. Vestibulum diam. Aliquam pellentesque, augue quis sagittis posuere, turpis lacus congue quam, in hendrerit risus eros eget felis. Maecenas eget erat in sapien mattis porttitor. Vestibulum porttitor. Nulla facilisi. Sed a turpis eu lacus commodo facilisis. Morbi fringilla, wisi in dignissim interdum, justo lectus sagittis dui, et vehicula libero dui cursus dui. Mauris tempor ligula sed lacus. Duis cursus enim ut augue. Cras ac magna. Cras nulla. Nulla egestas.

Curabitur a leo. Quisque egestas wisi eget nunc. Nam feugiat lacus vel est. Curabitur consectetur.

Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.

A subsection

Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor. Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus.

$$\left\| \begin{array}{ccccccc} \frac{\varphi}{(\varphi_1, \varepsilon_1)} & 0 & \dots\dots\dots & 0 \\ \frac{\varphi k_{21}}{(\varphi_2, \varepsilon_1)} & \frac{\varphi}{(\varphi_2, \varepsilon_2)} & 0 & \dots\dots\dots & 0 \\ \frac{\varphi k_{31}}{(\varphi_3, \varepsilon_1)} & \frac{\varphi k_{32}}{(\varphi_3, \varepsilon_2)} & \frac{\varphi}{(\varphi_3, \varepsilon_3)} & 0 & \dots\dots\dots & 0 \\ \vdots & & & \ddots & & \vdots \\ \frac{\varphi k_{n-21}}{(\varphi_{n-2}, \varepsilon_1)} & \frac{\varphi k_{n-22}}{(\varphi_{n-2}, \varepsilon_2)} & \dots\dots\dots & \frac{\varphi k_{n-2n-3}}{(\varphi_{n-2}, \varepsilon_{n-3})} & \frac{\varphi}{(\varphi_{n-2}, \varepsilon_{n-2})} & 0 & 0 \\ \frac{\varphi k_{n-11}}{(\varphi_{n-1}, \varepsilon_1)} & \frac{\varphi k_{n-12}}{(\varphi_{n-1}, \varepsilon_2)} & \dots\dots\dots & & \frac{\varphi k_{n-1n-2}}{(\varphi_{n-1}, \varepsilon_{n-2})} & \frac{\varphi}{(\varphi_{n-1}, \varepsilon_{n-1})} & 0 \\ \frac{\varphi k_{n1}}{(\varphi_n, \varepsilon_1)} & \frac{\varphi k_{n2}}{(\varphi_n, \varepsilon_2)} & \dots\dots\dots & & & \frac{\varphi k_{nn-1}}{(\varphi_n, \varepsilon_{n-1})} & \frac{\varphi}{(\varphi_n, \varepsilon_n)} \end{array} \right\| \quad (1.2)$$

1.2 Description our paradigm [8]

Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Donec odio elit, dictum in, hendrerit sit amet, egestas sed, leo. Praesent feugiat sapien aliquet odio. Integer vitae justo. Aliquam vestibulum fringilla lorem. Sed neque lectus, consectetur at, consectetur sed, eleifend ac, lectus. Nulla facilisi. Pellentesque eget lectus. Proin eu metus. Sed porttitor. In hac habitasse platea dictumst. Suspendisse eu lectus. Ut mi mi, lacinia sit amet, placerat et, mollis vitae, dui. Sed

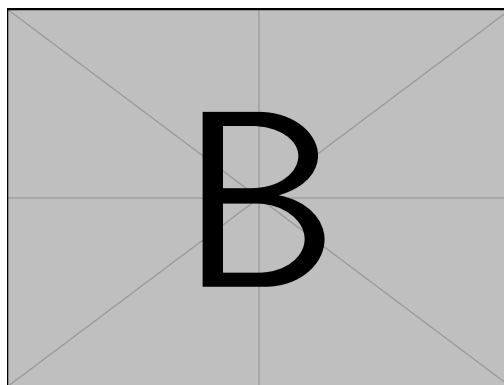


Figure 1.2: Caption text

ante tellus, tristique ut, iaculis eu, malesuada ac, dui. Mauris nibh leo, facilisis non, adipiscing quis, ultrices a, dui. No dissertation is complete without footnotes.^{1,2,3}

1.2.1 Conversion to a metaheuristic

Sed feugiat. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Ut pellentesque augue sed urna. Vestibulum diam eros, fringilla et, consectetur eu, nonummy id, sapien. Nullam at lectus. In sagittis ultrices mauris. Curabitur malesuada erat sit amet massa. Fusce blandit. Aliquam erat volutpat. Aliquam euismod. Aenean vel lectus. Nunc imperdiet justo nec dolor.

Etiam euismod. Fusce facilisis lacinia dui. Suspendisse potenti. In mi erat, cursus id, nonummy sed, ullamcorper eget, sapien. Praesent pretium, magna in eleifend egestas, pede pede pretium lorem, quis consectetur tortor sapien facilisis magna. Mauris quis magna varius nulla scelerisque imperdiet. Aliquam non quam. Aliquam porttitor quam a lacus. Praesent vel arcu ut tortor cursus volutpat. In vitae pede quis diam bibendum placerat. Fusce elementum convallis neque. Sed dolor orci, scelerisque ac, dapibus nec, ultricies ut, mi. Duis nec dui quis leo sagittis commodo. This concept is discussed further in section 1.4, and Refs. [18], [19].

1.3 Other generalizations

1.3.1 The most general case

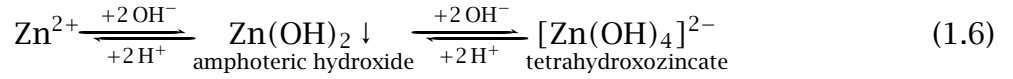
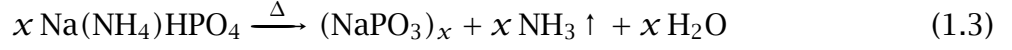
Sed commodo posuere pede. Mauris ut est. Ut quis purus. Sed ac odio. Sed vehicula hendrerit sem. Duis non odio. Morbi ut dui. Sed accumsan risus eget odio. In hac habitasse platea dictumst. Pellentesque non elit. Fusce sed justo eu urna porta tincidunt. Mauris felis odio, sollicitudin sed, volutpat a, ornare ac, erat. Morbi quis dolor.

¹First footnote. See section 1.4.

²Another interesting detail.

³And another really important idea to have in mind [12]-[17].

Donec pellentesque, erat ac sagittis semper, nunc dui lobortis purus, quis congue purus metus ultricies tellus. Proin et quam. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Praesent sapien turpis, fermentum vel, eleifend faucibus, vehicula eu, lacus. And another citation, so that our sources will be unambiguous [20].



These examples of chemical formulæ are copied directly from the documentation of the mhchem package, which was used to typeset them.

1.4 Baroclinic generation of vorticity

Substitution of the particle acceleration and application Stokes theorem leads to the *Kelvin-Bjerknes circulation theorem*, for $\rho \neq \text{fn}(p)$:

$$\frac{d\Gamma}{dt} = \frac{d}{dt} \int_C \mathbf{u} \cdot d\mathbf{r} \quad (1.7)$$

$$= \int_C \frac{D\mathbf{u}}{Dt} \cdot d\mathbf{r} + \underbrace{\int_C \mathbf{u} \cdot d\left(\frac{d\mathbf{r}}{dt}\right)}_{=0} \quad (1.8)$$

$$= \iint_S \nabla \times \frac{D\mathbf{u}}{Dt} \cdot d\mathbf{A} \quad (1.9)$$

$$= \iint_S \nabla p \times \nabla \left(\frac{1}{\rho} \right) \cdot d\mathbf{A} \quad (1.10)$$

Baroclinic generation of vorticity accounts for the sea breeze and various other atmospheric currents in which temperature, rather than pressure, creates density gradients. Further, this phenomenon accounts for ocean currents in straits joining more and less saline seas, with surface currents flowing from the fresher to the saltier water and with bottom current going oppositely.

Nomenclature for Chapter 1

Roman letters

C	material curve
\mathbf{r}	material position [m]
\mathbf{u}	velocity [m s^{-1}]

Table 1.1: The error function and complementary error function

x	$\text{erf}(x)$	$\text{erfc}(x)$	x	$\text{erf}(x)$	$\text{erfc}(x)$
0.00	0.00000	1.00000	1.10	0.88021	0.11980
0.05	0.05637	0.94363	1.20	0.91031	0.08969
0.10	0.11246	0.88754	1.30	0.93401	0.06599
0.15	0.16800	0.83200	1.40	0.95229	0.04771
0.20	0.22270	0.77730	1.50	0.96611	0.03389
0.30	0.32863	0.67137	1.60	0.97635	0.02365
0.40	0.42839	0.57161	1.70	0.98379	0.01621
0.50	0.52050	0.47950	1.80	0.98909	0.01091
0.60	0.60386	0.39614	1.8214	0.99000	0.01000
0.70	0.67780	0.32220	1.90	0.99279	0.00721
0.80	0.74210	0.25790	2.00	0.99532	0.00468
0.90	0.79691	0.20309	2.50	0.99959	0.00041
1.00	0.84270	0.15730	3.00	0.99998	0.00002

Greek letters

Γ circulation [$\text{m}^2 \text{s}^{-1}$]

ρ mass density [kg m^{-3}]

Appendix A

Code listing

```
1 function print_rate(kappa,xMin,xMax,npoints,option)
2     local c = 1-kappa*kappa
3     local croot = (1-kappa*kappa)^(1/2)
4     local logx = math.log(xMin)
5     local psi = 0
6
7     local xstep = (math.log(xMax)-math.log(xMin))/(
8         npoints-1)
9
10    arg0 = math.sqrt(xMin/c)
11    psi0 = (1/c)*math.exp((kappa*arg0)^2)*(erfc(
12        kappa*arg0)-erfc(arg0))
13
14    if option~= [[]] then
15        tex.sprint("\\addplot+[\"..option..\"]
16            coordinates{")
17        -- addplot+ for color cycle to work
18    else
19        tex.sprint("\\addplot+ coordinates{")
20    end
21    tex.sprint("("..xMin..","..psi0..")")
22
23    for i=1, (npoints-1) do
24        x = math.exp(logx + xstep)
25        arg = math.sqrt(x/c)
26        karg = kappa*arg
27        if karg<5 then
28            -- this break compensates for exp(karg^2),
29            -- which multiplies the error in the erf
30            -- approximation...
31            logpsi = -math.log(croot) + karg^2 +
32                math.log(erfc(karg)-erfc(arg))
33            psi = math.exp(logpsi)
34        else
```

```

29         psi = (1/(karg) - 1/(2*(karg^3)) + 3/(4*(
           arg^5))) /(1.77245385*croot)
30         -- this is the large x asymptote of the
           reaction rate
31     end
32     logx = math.log(x)
33     tex.sprint("(" .. x .. ", " .. psi .. ")")
34 end
35 tex.sprint("}")
36 end
37 \end{luacode*}

```

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